On the incidence of a financial transactions tax in a model with fire sales

Felix Bierbrauer*

University of Cologne, Germany

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Abstract

This paper studies the impact of a financial transactions tax on a financial market where financial institutions trade with each other. The motive for trade are differences in liquidity needs. Some financial institutions have to sell assets to generate the liquidity that is needed to meet their short-term obligations. Assets are marked to the market and financial institutions with negative equity are forced out of business. We show that the introduction of a financial transactions tax may lead to an increase of the number of distressed financial institutions and is therefore problematic from the viewpoint of financial stability.

Keywords: Financial transactions tax, financial stability, financial markets, cash-in-the-market-pricing, marking-to-market.

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1 Introduction

The 2008/2009 financial crisis has generated a demand for new taxes to be raised in the financial sector. The objective is to compensate tax payers for the costs of the financial crisis: “At their September 2009 Pittsburgh Summit, G20 Leaders requested the International Monetary Fund to prepare a report on how the financial sector could make a fair and substantial contribution to meeting the costs associated with government interventions to repair it (IMF, 2010)”. Of the various alternatives considered by the IMF (which included bank levies, that is, insurance premia for bailout subsidies, and a financial activities tax, a tax on income generated in the financial sector), one has since received particular interest of policy makers and the public at large, namely a financial transactions tax. In September 2011, the European Commission proposed a EU-wide financial transactions tax (European Commission, 2011). The proposal is controversial. The British government opposes it, the German government supports it, and emphasizes the need of an EU-wide tax, the French government also supports it, and, moreover, is willing to introduce it as a national tax should an EU-wide tax prove to be politically infeasible.

For a public finance economist this political controversy is a challenge. A public finance economist is supposed to be a specialist on questions like “how does a market outcome change if a tax is introduced”, or “who is ultimately carrying the burden of a tax”. In the context of notoriously fragile financial markets another question is of importance: How is “financial stability” affected by a tax on financial transactions. Yet, public finance as a field has very little to say on how financial market outcomes are affected by taxation.1 This would not matter so much if economists who specialize on banks and financial markets could substitute for this lack of knowledge. However, taxes are not among the policy instruments which have attracted their attention in the past. Instead they have been focussing on regulatory tools (such as equity requirements, limits on leverage, restrictions on risk-taking) or on the policies that are used to stabilize financial markets in periods of systemic crises (bail-outs, liquidity support, purchases of toxic assets by the government).2

This paper is a first attempt to fill this gap. It studies a market in which financial institutions trade with each other and asks how endogenous quantities such as the price

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1There is research on the treatment of financial services under a value added tax system, see Huizinga (2002), Auerbach and Gordon (2002), or Genser and Winker (1997). Lockwood (2010) studies the optimal taxation of financial services in a dynamic Ramsey model. Philippon (2010) characterizes the optimal size of the financial sector relative to the “real” economy in an endogenous growth model. In his model, taxes can be used to improve upon the relative sizes of these sectors that would result under laissez-faire. What all these articles have in common is that they model the relation between a financial intermediary and agents who demand financial services. They do not model financial markets, i.e. markets where financial institutions trade financial products with each other.

2Some recent papers study Pigouvian taxation as a response to systemic risk-taking (Acharya e.a., 2010), maturity mismatch (Perotti and Suarez, 2011), or excessive leverage (Keen, 2011). Again, these papers do not model financial markets in an explicit way.
on such a market, the volume of trade and, most importantly, the number of distressed financial institutions are affected by the introduction of a financial transactions tax.

The main result is the following: A financial transactions tax drives a wedge between the price that the buyer of an asset pays and the price that the seller receives. Now, consider a distressed financial institutions that has a liquidity problem and therefore needs to sell assets in order to survive. The financial transactions tax makes it more difficult for such an institution to avoid bankruptcy because it has to sell more assets to a generate a given volume of liquidity. Formally, we will show that the equilibrium that results after a financial transactions tax has been introduced has lower prices for banks that sell assets and more bankruptcies among those that have to sell in order to survive. This increase in bankruptcies need not be small. The analysis may give rise to multiple equilibria. There may coexist an equilibrium that has high prices and is therefore good from a financial stability viewpoint because it involves few bankruptcies, and a low-price or bad equilibrium. The introduction of a financial transactions tax may eliminate the good equilibrium and therefore lead to a drastic price decrease accompanied by a drastic increase of financial distress. To sum up, from the perspective of financial stability, a financial transactions tax has problematic consequences.

The formal analysis is based on a model that borrows essential features from the work of Allen and Gale (1994, 2004a,b). There are three periods, $T \in \{0, 1, 2\}$, and a large number of banks. In the initial period, each bank obtains funds from risk-averse debt-holders and risk-neutral providers of equity. A financial institution, henceforth simply a bank, seeks to maximize its expected return on equity and has to respect a participation constraint for debt-holders. In the initial period it chooses a debt structure, that is, promises to debt-holders that are due in $T = 1$ and $T = 2$, respectively, and it decides how to invest its funds. It can either invest in projects that mature in $T = 1$ or in projects that mature in $T = 2$. Both investments are risky. It becomes known in $T = 1$ whether investments are successful or fail. There is both aggregate and idiosyncratic uncertainty: The performance of the investments of an individual bank, as well as the cross-section distribution of performance are ex ante unknown.

A long-term investment cannot be liquidated, but a claim on the cash flow that it generates, henceforth simply an asset, can be sold in $T = 1$ on a financial (spot) market. The motive for trade is that banks differ in their liquidity needs. After the state of the world has been revealed in $T = 1$, some banks find that they have more liquidity than they need in order to honor their current obligations and are therefore willing to buy other banks’ assets at an appropriate price. Other banks do not have enough liquidity and therefore need to sell assets. Moreover, there is cash-in-the-market-pricing, i.e., it is possible that there are only few banks who are capable of buying assets. Put differently, there are only few banks who have more cash than they need and this shortage of cash

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may depress the price which equates demand and supply on the financial market.

We model financial distress via a bankruptcy regime. A bank is bankrupt, if, at the given financial market price, the value of its liabilities exceeds the value of its assets. In case of bankruptcy a bank is liquidated and its assets are sold on the financial market. In our basic model, we do not consider bank-bailouts, as in Acharya and Yorulmazer (2008). Instead we assume that, in case of bankruptcy, a bank’s debt-holders are bailed out by the government, and that equity is wiped out. As an extension, we also consider the case in which the government purchases the assets of distressed financial institutions.

The analysis of this model yields the following insights: There is generally not a unique financial markets equilibrium. The reason is that some banks may find themselves in \( T = 1 \) in the situation of a fire-seller: They need to sell some of their assets to generate the liquidity that is needed in order to be able to honor their short-term debt. The supply of assets of these banks will be a downward-sloping function of the price they can reap: if the price is high they need to sell only few asset, and they need to sell many assets if the price is high. This creates a possibility of the coexistence of “low-price equilibria” in which a lot of assets need to be sold, and a lot of banks go bankrupt and a “high-price equilibria” with the opposite characteristics.

The equilibrium multiplicity poses a difficulty for an analysis of how the equilibrium changes if a financial transactions tax is introduced. However, if we limit attention to equilibria that satisfy a regularity condition, then a robust finding is available: A small tax leads to a reduction of the price for sellers and (weakly) increases the number of fire-selling banks that cannot avoid bankruptcy. However, it is not possible to come up with a robust finding for how the price that buyers pay is affected, this price may go up, it may go down or it may remain constant. This implies, in particular, that there is also no general conclusion of how financial stability, as measured by the total number of banks that go bankrupt, is affected by the introduction of a financial transactions tax. The reason is that, in addition to distressed fire-selling banks, there may be banks that have enough liquidity in \( T = 1 \), but experience drastic failures of their long-term investments. These banks need to buy assets if they want to be able to honor their obligations to debt-holders in \( T = 2 \). We refer to such banks as fire-buyers. Hence, if the price for buyers falls in response to an increase of the tax rate, more fire-buyers will be able to avoid bankruptcy. An assessment of the impact of a financial transactions tax then has to trade-off the reduced number of bankrupt fire-buyers against an increased number of bankrupt fire-sellers.

To be able to offer more definite conclusions on the impact of a financial transactions tax, we refer to Allen and Barletti (2012) and Cifuentes et al. (2015).
tax we then turn to interesting special cases of the general setup. As a first benchmark, we study an economy that has no fire-selling banks, i.e. we assume that – possibly as a result of regulation – in $T = 1$ all banks have enough liquidity to honor their current obligations, but some experience problems with their long term investments. For such an economy we can show that a financial transactions tax has no impact at all on the financial market equilibrium. All banks are either safe or fire-buyers, and the supply on the financial market is entirely due to the fire-buying banks that fail. We show that all these quantities are functions of the price that buyers face on the financial market, and remain unaffected if a financial transactions tax drives a wedge between the price that buyers pay and the price that sellers get. The tax only reduces the liquidation values for the failed banks’ debt-holders, but since they are assumed to be bailed-out by the government this makes no difference for them. It also makes no difference for the government. The increase in tax revenue is exactly matched by the increase of the bail-out payments for debtors.

As our main application of interest, we then consider an economy with the following features: There is idiosyncratic liquidity risk, so that some banks have more liquidity than they need in $T = 1$ and others do not have enough of it. In addition, there is aggregate risk with respect to the performance of long-term investments. This aggregate risk affects all banks in the same way, which is meant to be descriptive of the 2008/2009 financial crisis in which many banks were exposed in similar ways to the performance of subprime mortgages. Finally, we assume that banks have only little long-term debt, which implies that there will be no fire-buyers, simply because their are no long-term obligations that need to be honored. The assumption captures the reliance of many banks on short-term financing via the repo or the commercial paper market. Under these assumptions, we obtain clear-cut results on the impact of a financial transactions tax: the price that sellers get falls, the price that buyers pay does not go up, and the number of bankruptcies rises. These changes need not be small. If there are multiple equilibria, then the introduction of the tax may eliminate the “good” one that has only few bankruptcies and induce a jump into the “bad” one with a drastic fall in prices and a drastic increase in the number of bankruptcies. We also study the impact of various policies (liquidity support, government purchases of assets, solvency regulation) in this model and show that they would indeed have a stabilizing impact. We then show that the stabilizing impact of these policy measures may be neutralized by the introduction of a financial transactions tax.

The remainder is organized as follows. The next section lays out a general model. In Section 3 we discuss equilibrium existence and comparative statics for this general framework. Section 4 contains the analysis of an economy without fire sales. Section 5 deals with the analysis of an economy with idiosyncratic liquidity risk and aggregate long-term investment risk. The last section concludes. All proofs are in an Appendix.
2 The Model

There is a continuum of banks of measure 1. The set of banks is denoted by $I$, with typical element $i$. There are three dates, $T \in \{0, 1, 2\}$. Bank $i$ is endowed with equity $e_i$ and debt $d_i$. It promises its debtors payments of $x_{i1}$ in period 1 and of $x_{i2}$ in period 2. We can think of debtors either as depositors as in a model of liquidity insurance, or as lenders in the commercial paper or repo market.\(^5\) Holders of equity receive the residual value of the bank’s assets (i.e. what is left after the payments promised to depositors and other debtors are made) in case there is no failure. Hence, bank $i$ has funds of

$$a_i := d_i + e_i$$

to invest in $T = 0$. Bank $i$ lends out or invests an amount $y_{i1}^0$ with a short term perspective. These investments are risky. They fail with a certain probability, in which case they return 0 in $T = 1$. If they do not fail, they return $r_1$ in $T = 1$. For ease of notation, we let $r_1 = 1$ in the following. In addition, bank $i$ lends out an amount of $y_{i2}^0 = a_i - y_{i1}^0$ with a long run perspective. We think of these investments as risky loans to entrepreneurs, home-owners etc. A risky loan fails with a certain probability, in which case it returns 0. If it does not fail, it returns $r_2$ in $T = 2$. Whether or not a loan performs becomes known in $T = 1$. For now, we treat $e_i, d_i, x_{i1}, x_{i2}, y_{i1}^0$ and $y_{i2}^0$ as exogenous.

Long-run investments can not be liquidated; that is, it is not possible to withdraw in $T = 1$ the funds invested in $T = 0$. However, it is possible to trade claims on the returns of these investments on a financial market in $T = 1$. The financial market equilibrium is described in more detail below. We also assume that, in $T = 1$, banks have access to a riskless storage technology in order to transfer cash available in $T = 1$ into cash available in $T = 2$.

We assume that a bank’s objective is to maximize the expected return on equity, which we denote by $E[\pi^2_i]$. We assume, for simplicity, that the holders of equity require a payment only in $T = 2$, and $\pi^1_i$ is the payment per unit of equity in $T = 2$. As of $T = 0$, $\pi^2_i$ is a random quantity. It depends both on idiosyncratic risk (such as the fraction of successful investments of bank $i$) and aggregate risk (such as the price on the financial market), which is realized in $T = 1$.

State of the economy. In $T = 1$, each bank $i$ is hit by a shock. Formally, in $T = 1$, bank $i$ is characterized by a vector $\sigma_i = (y_{i1}, y_{i2})$, where $y_{i1} \leq y_{i1}^0$ is the fraction of performing short-run investments, and $y_{i2} \leq y_{i2}^0$ is the fraction of performing long-run investments, i.e., of investments that will yield a return in $T = 2$. From the perspective

\(^5\)Consider a model with depositors who demand liquidity insurance and suppose that depositors are promised $c_{i1}$ if they withdraw early and $c_{i2}$ if they withdraw late. Then $x_{i1} = s_i c_{i1}$ and $x_{i2} = (1 - s_i) c_{i2}$, where $s_i$ is the fraction of early consumers among the depositors of bank $i$. 

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of the initial period, \( \sigma_i \) is a random quantity. As of \( T = 0 \), bank \( i \) does not yet know how many of its long- and short-run investments will perform, implying that \( y_{i1} \) and \( y_{i2} \) are random quantities. We assume, for simplicity, that the banks’ liabilities \( x_{i1} \) and \( x_{i2} \) are not random.\(^6\)

In the following we refer to the collection \( \sigma = (\sigma_i)_{i \in I} \) as the state of the economy.\(^7\)

There is a financial market where funds available in \( T = 1 \) can be traded against the long-run investments. That is, while the investments made in the initial period cannot be liquidated in \( T = 1 \) they can be sold on a financial market, for instance, to generate the liquidity that may be needed in order to honor promises which are due in \( T = 1 \). We view the price on the financial market as a function of the state of the economy. Moreover, we consider the possibility that trades on the financial market are subject to an (ad valorem) transactions tax \( t \), which drives a wedge between the revenue from selling a loan, \( p(\sigma) \), and the cost of buying a loan \( q(\sigma) \), so that

\[
q(\sigma) = (1 + t)p(\sigma).
\]

In the reminder of this section, we treat the state of the economy as fixed. To save on notation, we will therefore suppress the dependence of the prices \( p \) and \( q \) on the state of the economy.

**Bankruptcies.** We assume that all economic agents have access to a storage technology in order to transfer resources available in \( T = 1 \) into resources available in \( T = 2 \). If instead of storing the resources available in \( T = 1 \), a bank buys assets on the financial market it receives \( r_2/q \) per unit invested. Hence, there will be demand on the financial market only if \( r_2 \geq q \).

We assume that a bank goes bankrupt if, at the given price on the financial market, the value of its liabilities is larger than the value of its assets. To clarify the conditions under which this is the case the following terminology is useful: Bank \( i \)'s assets in \( T = 1 \) are its endowment with cash \( y_{i1} \) and assets \( y_{i2} \). Since each asset generates \( r_2 \) units of resources available in \( T = 2 \), we can as well think of bank \( i \) as having an endowment of \( y_{i1} \) resources available in \( T = 1 \) and of \( r_2y_{i2} \) resources available in \( T = 2 \). It will prove convenient to also define the net endowments \( \theta_{i1} = y_{i1} - x_{i1} \) and \( \theta_{i2} = r_2y_{i2} - x_{i2} \), respectively.

If there was no financial transactions tax so that \( q = p \), it would be straightforward to determine whether or not a bank is bankrupt: Since \( p \) is the price of a performing loan and a performing loan generates \( r_2 \) units of consumption in \( T = 2 \), it has to be the case

\(^6\)In a model of liquidity insurance, this is the case if \( s_i \) is a known parameter, so that it is known a priori how many early consumers will show up in \( T = 1 \).

\(^7\)We do not make an assumption that, for any pair of banks \( i \) and \( j \), \( \sigma_j \) and \( \sigma_i \) are independent random variables. Also we do not (yet), impose restrictions on the possible realizations of the cross-section distribution of \( \sigma_i \), as would be implied by a “law of large numbers for large economies.” Thus, the model features both idiosyncratic and aggregate uncertainty.
that, as of $T = 1$, the price of a unit of consumption available in $T = 2$, equals $p/r_2$. Hence, bank $i$ goes bankrupt if

$$\theta_{i1} + \frac{p}{r_2}\theta_{i2} < 0,$$

and stays in business otherwise. With a financial transactions tax, however, there are two prices, one which is relevant if assets are sold on the financial market and one which is relevant if assets are bought. Which of these prices is relevant for the survival of bank $i$ depends on its net endowment.

Safe and failed banks. If $\theta_{i1} \geq 0$ and $\theta_{i2} \geq 0$ bank $i$ is safe in that its ability to honor its promises does not depend on market conditions. If $\theta_{i1} \leq 0$ and $\theta_{i2} \leq 0$, with at least one inequality being strict, bank $i$ fails irrespective of market conditions. In the following we denote the set of safe and failed banks by $I^*$ and $I^\dagger$, respectively.

Fire-purchases. Suppose that $\theta_{i1} > 0$ and $\theta_{i2} < 0$, so that bank $i$ has more resources than it needs in $T = 1$, but that it does not have enough resources in $T = 2$. Therefore, it will have to use its excess liquidity in $T = 1$ to buy assets on the financial market and the relevant price is $q$. The bank will therefore survive provided that

$$\theta_{i1} + \frac{q}{r_2}\theta_{i2} \geq 0,$$

or, equivalently, provided that

$$\frac{q}{r_2} \leq -\frac{\theta_{i1}}{\theta_{i2}}.$$

Note, in particular, that, for a bank that needs to buy assets, lower prices on the financial market make it easier to survive.\footnote{The bank could also use the storage technology, in which case it would be able to honor its promises provided that $\theta_{i1} + \theta_{i2} \geq 0$. However, since $\frac{q}{r_2} \leq 1$, it is more difficult to survive using the storage technology.} In the following, we denote by

$$I_{buy}^*(q) = \left\{ i \in I \mid \theta_{i1} > 0, \theta_{i2} < 0, \frac{q}{r_2} \leq -\frac{\theta_{i1}}{\theta_{i2}} \right\}$$

the set of banks that have to buy assets on the financial market and manage to survive at a price of $q$. Analogously, we denote by

$$I_{buy}^\dagger(q) = \left\{ i \in I \mid \theta_{i1} > 0, \theta_{i2} < 0, \frac{q}{r_2} > -\frac{\theta_{i1}}{\theta_{i2}} \right\}$$

the complementary set of banks that fail.

Fire-sales. Now suppose instead that $\theta_{i1} < 0$ and $\theta_{i2} > 0$. This bank has a liquidity problem in $T = 1$ and therefore needs to sell assets on the financial market. The relevant price is now $p$, and the bank survives provided that

$$\theta_{i1} + \frac{p}{r_2}\theta_{i2} \geq 0,$$
or, equivalently, provided that
\[ \frac{p}{r_2} \geq -\frac{\theta_{i1}}{\theta_{i2}}. \]

Note that, for a bank that needs to sell assets, lower prices on the financial market make it more difficult to survive.\(^9\) We denote by
\[ I^*_\text{sell}(p) = \left\{ i \in I \mid \theta_{i1} < 0, \theta_{i2} > 0, \frac{p}{r_2} \geq -\frac{\theta_{i1}}{\theta_{i2}} \right\} \]
the set of banks that have to sell assets on the financial market and manage to survive at a price of \( p \). Analogously, we denote by
\[ I^\dagger_\text{sell}(p) = \left\{ i \in I \mid \theta_{i1} < 0, \theta_{i2} > 0, \frac{p}{r_2} < -\frac{\theta_{i1}}{\theta_{i2}} \right\} \]
the complementary set of banks that fail.

A bank that is bankrupt gets liquidated. That is, its assets are sold on the financial market. The bank’s debtors then receive the proceeds from these sales plus the cash that the bank has available in \( T = 1 \). In case of bankruptcy, equity holders do not receive anything.

**Behavior of non-bankrupt banks.** Banks that do not go bankrupt can engage in trade on the financial market. Denote by \( z_i = (z_{i1}, z_{i2}) \) a portfolio consisting of cash available in \( T = 1 \) and holdings of the asset for bank \( i \). Bank \( i \) solves the following problem: Choose \( z_i \) in order to maximize the return to equity \( \pi^2_i \) subject to the following set constraints:

(i) The value of the portfolio \( z_i \) does not exceed the value of the endowment \( y_i = (y_{i1}, y_{i2}) \). If \( z_{i2} \geq y_{i2} \), so that bank \( i \) is a net buyer of assets on the financial market this requires that
\[ z_{i1} - y_{i1} + q(z_{i2} - y_{i2}) \leq 0. \] \(^{(1)}\)

If \( z_{i2} < y_{i2} \), then
\[ z_{i1} - y_{i1} + p(z_{i2} - y_{i2}) \leq 0. \]

(ii) It is possible to honor the promises in \( T = 1 \), \( z_{i1} \geq x_{i1} \).

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\(^9\)One sometimes reads about “fire sale externalities”. What is meant is that a larger supply on the financial market depresses the price and makes it more difficult to survive for a bank that has to engage in fire sales.
(iii) The holders of equity receive what is left after bank $i$ has honored its obligations in $T = 1$ and $T = 2$. Hence,

$$ e_i \pi_i^2 = r_2 z_{i2} - x_{i2} + z_{i1} - x_{i1} , $$

where $z_{i1} - x_{i1} > 0$ whenever excess liquidity from period 1 is stored until $T = 2$. The holders of equity are protected by limited liability, or, equivalently, by the bankruptcy regime. Hence,

$$ e_i \pi_i^2 \geq 0 , $$

which implies, in connection with (ii), that the bank has enough resources to honor its obligations in $T = 2$.

The solution to this problem depends on whether a bank is safe, and therefore faces a real choice between buying and selling assets on the financial market, or is a fire-seller of assets because of a lack of liquidity in $T = 1$, or is a fire-buyer in need of assets that generate resources in $T = 2$.

**Safe banks.** A safe bank has a choice between buying and selling assets on the financial market in $T = 1$. Since $r_2 \geq q(\sigma) \geq p(\sigma)$, selling an asset on the financial market and storing the proceeds until $T = 2$ reduces what can be paid out to equity-holders in $T = 2$. A safe bank will therefore not make use of the possibility to sell assets on the financial market if $r_2 > q(\sigma)$ or if $q(\sigma) > p(\sigma)$, and the relevant budget constraint is the one in (1). This constraint will obviously bind implying that

$$ z_{i1} = y_{i1} - q(z_{i2} - y_{i2}) . $$

Substituting this expression into the objective function in (2) yields

$$ e_i \pi_i^2 = r_2 z_{i2} - x_{i2} + y_{i1} - q(z_{i2} - y_{i2}) - x_{i1} = (r_2 - q)z_{i2} + qy_{i2} - x_{i2} + y_{i1} - x_{i1} . $$

Hence, if $r_2 > q$, a safe bank chooses $z_{i2}$ as large as possible and $z_{i1}$ as small as possible. The optimal portfolio is therefore

$$ z_i = \left( x_{i1}, y_{i2} + \frac{\theta_{i1}}{q} \right) . $$

If $r_2 = q$, by contrast, any portfolio $z_i$ satisfying $z_{i1} \geq x_{i1}$ and the budget constraint in (1) is optimal. The same is true if $r_2 = q = p$. A safe bank’s net demand demand on the financial market is therefore given by

$$ z_{i2} - y_{i2} = \begin{cases} \frac{\theta_{i1}}{q}, & \text{if } r_2 > q \\ \in [-y_{i2}, \frac{\theta_{i1}}{q}], & \text{if } r_2 = q . \end{cases} $$

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Surviving banks which have to fire-buy. A bank with $\theta_{i1} > 0$ and $\theta_{i2} < 0$ has to buy assets on the financial market in order to survive. Hence, $q$ is the relevant price and the optimizing behavior is like the one of a safe bank. A fire-buying bank’s demand on the financial market is therefore also given by (3).

Surviving banks which have to fire-sell. A bank with $\theta_{i1} < 0$ and $\theta_{i2} > 0$ has to sell assets on the financial market in order to survive. The relevant price is therefore $p$ which yields the following net demand of assets on the financial market

$$z_{i2} - y_{i2} = \begin{cases} \frac{\theta_{i1}}{p}, & \text{if } r_2 > p \\ \in [-y_{i2}, \frac{\theta_{i1}}{p}], & \text{if } r_2 = p. \end{cases}$$

Note that, if $r_2 > p$, the excess demand of fire-selling bank is negative,

$$z_{i2} - y_{i2} = \frac{\theta_{i1}}{p} < 0,$$

which means that the bank is adding to the supply of assets on the financial market.

Financial market equilibrium. Demand. The net demand for assets on the financial market stems from the safe banks and the banks who have to buy assets and manage to survive. Recall, that all these banks have more liquidity than they need in $T = 1$, so that $\theta_{i1} > 0$. Given a price of $q(\sigma) \in (0, r_2)$, the demand on the financial market is given by

$$D(q) := \int_{\hat{I}^{*}(q)} (z_{i2} - y_{i2}) \, di = \int_{I^{*}(q)} \frac{\theta_{i1}}{q} \, di$$

where

$$\hat{I}^{*}(q) = I^{*} \cup I^{*}_{buy}(q)$$

is the set of banks that demand assets on the financial market. For $q = r_2$ the demand is bounded from above by $D(r_2)$ and from below by

$$-\int_{I^{*}(q)} y_{i2} \, di.$$

To see this, note that if $q = r_2$, these banks are indifferent between storing their cash and using it to buy loans on the financial market. Consequently, as a group they are willing to hold any amount of assets that lies between 0, in which case their net demand equals $-\int_{I^{*}(q)} y_{i2} \, di$ and $\int_{I^{*}(q)} y_{i2} \, di + D(r_2)$. Observe that the demand function $D$ is downward-sloping for $q \in (0, r_2)$. If $q$ goes up this reduces the demand of each bank in $\hat{I}^{*}(q)$. In addition, the set $I^{*}_{buy}(q)$ and hence the set $\hat{I}^{*}(q)$ shrinks, since, for banks that have to buy, a higher price makes it more difficult to survive.

Supply. The net supply on the financial market has two sources: First, the assets of all banks who fail are sold. Second, some banks fire-sell to generate liquidity and thereby
manage to survive. For these banks we have that $\theta_{i1} < 0$. Therefore, for $p \in (0, r_2)$ the supply on the financial market equals

$$\bar{S}(p) := \int_{\hat{I}(p)} y_{i2} \, di - \int_{I_{\text{sell}}^*(p)} \frac{\theta_{i1}}{p} \, di ;$$

where

$$\hat{I}(p) := I^1 \cup I_{\text{buy}}^1(p(1 + t)) \cup I_{\text{sell}}^1(p)$$

In this formula, we view the supply on the financial market as a function of the revenue $p$ that a seller can realize. Since $q = p(1 + t)$ we could as well interpret market supply as a function of $q$. We define

$$S(q) := \bar{S}\left(\frac{q}{1 + t}\right)$$

as the financial market supply as a function of the price $q$ that buyers have to pay.

For $p = r_2$, $\bar{S}(r_2)$ is only a lower bound on the supply on the financial market, since, at this price, the surviving fire-sellers would be willing to sell all their assets. The corresponding upper bound is

$$\int_{\hat{I}(r_2) \cup I_{\text{sell}}^*(r_2)} y_{i2} \, di .$$

Without further assumptions, the slope of the supply function cannot be determined. If $p$ goes up, this implies that the set $I_{\text{sell}}^1(p)$ shrinks, which tends to reduce the supply, but the set $I_{\text{sell}}^*(p)$ expands, which tends to increase it. Moreover, for each bank in $I_{\text{sell}}^*(p)$ the number of assets that are put on the market goes down. Finally, $I_{\text{buy}}^1(p(1 + t))$ increases because it gets more difficult to survive as a buyer. The interplay of all these effects imply that the slope of the supply function can not be signed a priori.

**Equilibrium.** Let $q \in (0, r_2)$. We say that $q$ is an equilibrium price if

$$D(q) = S(q) .$$

If $q = r_2$, then $q$ is an equilibrium price if

$$D(q) \geq S(q) .$$

Why is $q = r_2$ an equilibrium price even if $D(r_2) > S(r_2)$? The reasoning is as follows: At $q = r_2$, all non-bankrupt banks are indifferent between storing a unit of cash until $T = 2$, and using it to buy $\frac{1}{q}$ assets on the financial market, for a return of $r_2 \frac{1}{q} = 1$ in $T = 2$. Therefore, $D(r_2)$ is to be interpreted as an upper bound on the amount of assets that the buying banks are willing to hold. At this price, they are willing to hold any number of assets between 0 and $D(r_2)$. If $D(r_2) > S(r_2)$ it is therefore possible to arrange trade between buyers and sellers so that, at the given price of $q = r_2$, every non-bankrupt bank gets a profit-maximizing portfolio.
Figure 1: A financial market equilibrium such that demand and supply do not intersect. The unique equilibrium price is $q^* = r_2$.

3 How do financial market equilibria change if a financial transactions tax is introduced?

In this section, we provide a comparative statics analysis of how financial market equilibria change if a financial transactions tax is introduced. As a preliminary step we note that the set of equilibria is typically non-empty.

Proposition 1 Suppose that $D$ and $S$ are continuous functions on $[0, r_2]$, and that $\hat{I}^*(0) \neq \emptyset$. Then, an equilibrium price exists.

The Proposition asserts that a financial market equilibrium exists under very weak conditions. All that is needed, apart from continuity, is that at the minimal price of $q = 0$, demand exceeds supply. Recall that the net demand of a bank that has excess liquidity in $T = 1$ equals $\theta_1 q$. This grows without limits as $q$ converges to 0. Hence, what is needed for existence is that there exists some bank that has more cash than it needs to honor its obligations in $T = 1$.

Obviously, an equilibrium exists if the demand and the supply curve intersect over the range of admissible financial market prices $q \in (0, r_2]$. An equilibrium exists even if they do not intersect. This situation is illustrated in Figure 1. If there is no intersection, this means that, for all admissible $q$, demand exceeds supply in the sense that $D(q) > S(q)$. But this implies, in particular that $D(r_2) > S(r_2)$ so that $r_2$ is an equilibrium price.

It is also possible that there exist multiple equilibria as illustrated in Figure 2. This figure displays a linear and downward-sloping supply curve. In Section 5 we will look at a more specific version of this model, which makes it possible to show that the situation in Figure 2 arises under a set of “natural” assumptions.
then these banks have to sell a lot of assets to generate the liquidity that they need in
$T = 1$. If the price is high, the receive a lot of cash per asset which implies that the
number of assets that they need to sell on the market is reduced. Hence, low prices lead
to a large supply and high prices lead to a small supply on the financial market implying
that supply is a decreasing function of the price. This may lead to multiple equilibria.
In Figure 2, there are three equilibria: Two equilibria correspond to the intersections
of the supply and the demand curve. In addition, $q = r_2$ is an equilibrium price since
$D(r_2) > S(r_2)$.$^{11}$

The different equilibria cannot be Pareto-ranked. An equilibrium with a low price is
preferred by those on the demand side. In fact, $q < r_2$ means that they can buy assets
below their net present value and make a speculative profit. By contrast, for banks that
have to fire-sell equilibria with a high price are preferred. Higher prices makes it more
likely that they survive and that their shareholders receive a profit. If the government
provides protection to the banks’ debtors (via deposit insurance or bail-out guarantees)
it will prefer the equilibrium with the lowest number of bankruptcies. However, it is
not clear a priori that a higher financial market price is desirable from the government’s
perspective. This holds only if all banks at the brink of survival are fire-sellers. Fire-buyers
need low asset prices in order to survive.

**Comparative static effects of an increase of the financial transactions tax.** We
can illustrate the impact of an increased tax $t$ graphically. Recall that we first defined
supply on the financial market as a function of a seller’s per unit revenue $p$. This led to

$^{11}$One can easily verify that the number of equilibria has to be odd. If the demand and the supply curve
do not intersect there is one equilibrium. This is the situation in Figure 1. If they intersect once, then
the intersection is the unique equilibrium. If they intersect twice, as in Figure 2, there have to be three
equilibria. If they intersect three times, each intersection is an equilibrium as in Figure 3 below. If they
intersect four times, each intersection is an equilibrium and, in addition, there is the fifth equilibrium at
$q = r_2$, etc.
the supply function $\bar{S} : p \mapsto \bar{S}(p)$. To be able to draw the supply and the demand curve into the same diagram we than defined the supply function $S : q \mapsto S(q)$ with

$$S(q) = \bar{S} \left( \frac{q}{1+t} \right),$$

which gives the supply on the financial market as a function of the price $q$ that a buyer has to pay on the financial market. Now, if we change the tax from a level of $t$ to a level of $t' > t$ this yields a new supply functions $S_1$ with

$$S_1(q) = \bar{S} \left( \frac{q}{1+t'} \right).$$

Graphically, we can depict this as a shift of the supply curve to the right: A given supply is now associated with a higher price $q$ for the buyers in order to compensate for the fact that the sellers receive a smaller fraction of the buyer’s total spending. This is illustrated in Figure 3.\(^{12}\)

Figures 3 and 4 show how an increase of the tax rate $t$ can affect the set of equilibria. In Figure 3 there are initially three equilibria, namely the three intersections of the supply and the demand curve. After a rightward shift of the supply curve, there is only one intersection left. Hence, the increase of $t$ implies that we move from a situation with three equilibria to a situation with a unique equilibrium. In Figure 4, by contrast, the structure of the set of equilibria remains unaffected. Before and after the change of the tax rate, there are three equilibria.

Even if the structure of the set of equilibria remains unchanged as in Figure 4, it is not straightforward to determine whether the price on the financial market goes up or down in response to an increase of the financial transactions tax. If we focus on the “high-price equilibrium” or the “equilibrium in the middle” we would conclude that the increase of

\(^{12}\)Since we have modeled the financial transactions tax as an ad valorem tax the shift to the right is larger the larger is $q$
Figure 4: Comparative static effects of an increase of the financial transactions tax: Regular versus unregular equilibria.

\( t \) drives the financial market price for buyers, \( q \), up. The “low-price equilibrium”, by contrast, has a lower value of \( q \) after an increase of the tax rate. A typical approach (see Mas-Collel e.a. (1995)) is to view the “equilibrium in the middle” as irregular or unstable. The reasoning is as follows: If we assume that excess demand drives prices up, and excess supply drives prices down, then, starting from any non-equilibrium price, we well not approach the “equilibrium in the middle”, but either the “low-price equilibrium” or the “high-price equilibrium”. The “low-price equilibrium” and the “high-price equilibrium” are both stable or regular because the demand curve intersects the supply curve from above, so that demand exceeds supply for prices slightly smaller than the equilibrium price and supply exceeds demand for prices lightly larger than the equilibrium price. As the following Proposition shows, the focus on stable equilibria indeed makes it possible to offer one general conclusion:

**Proposition 2** Suppose that the functions \( D \) and \( S \) are both continuously differentiable and that there is an initial tax rate \( t_0 \). Consider an equilibrium that is not irregular, i.e. it is either such that \( D(q) = S(q) \) and \( D' < S' \), or it is such that the equilibrium price equals \( r_2 \) and \( D(r_2) > S(r_2) \). There exists \( \epsilon > 0 \) so that moving to a higher tax rate \( t_1 \in (t_0, t_0 + \epsilon) \) gives rise to a new equilibrium which is in the neighborhood of the old equilibrium and has the following properties:

i) The price that sellers receive goes down: \( p_1 < p_0 \), where \( p_1 \) and \( p_0 \) are the equilibrium prices for sellers before and after the tax change, respectively.

ii) For fire-selling banks it is more difficult to survive in the new equilibrium: \( p_1 < p_0 \) implies that \( I_{sell}^*(p_1) \subset I_{sell}^*(p_0) \).
The Proposition shows that there is a rather general comparative statics result: If we are in a regular equilibrium, then, an increase of the tax rate will depress the revenue $p$ that fire-selling banks realize per asset that they put on the financial market. As a consequence, it gets harder for them to generate the liquidity that is needed in order to survive and (weakly) more fire-sellers will go bankrupt.

However, even if we are prepared to focus on stable equilibria it is difficult to come up with more general statements on what happens on financial markets if a financial transactions tax is introduced. This can be illustrated with the help of Figure 4. If we focus on the “small-price equilibrium” in Figure 4, then the implications of a small increase of the tax rate are the following: The equilibrium price $q$ that buyers have to pay goes down. Hence, for fire-buying banks it is easier to survive in the new equilibrium since $q_1 < q_0$ implies that $I_{buy}(q_0) \subset I_{buy}(q_1)$. By contrast, if we focus on the “large-price-equilibrium” in Figure 4, we would get comparative statics results which are reminiscent of those from a conventional partial equilibrium analysis: Buyers have to pay more; i.e., $q$ goes up. Sellers get less; i.e., $p$ goes down. The equilibrium quantity goes down. Finally, for all banks – for fire-selling as well as for fire-buying banks – it gets more difficult to survive. Given these difficulties, we will look at various special cases of this model which make it possible to offer more definite conclusions. In Section 5 will look at a specific model with correlated long-run investment risk and idiosyncratic liquidity risk, which may be descriptive of the events following the collapse of Lehman brothers in September 2008. This model gives rise to fire sales, and we will then look at how various policy measures such as liquidity support and solvency regulation affect the incidence of a financial transactions tax. Before we turn to this specific model, we will, first look at the benchmark of an economy in which fire sales do not arise.

4 No fire sales

We defined a fire-selling bank by the property that $\theta_{i1} < 0$. This property implies that, in order to avoid bankruptcy, such a bank has to sell some of its assets on the financial market. The following assumption makes sure that fire-sales do not arise.

Assumption 1 For all $\sigma$ and all $i$, $\theta_{i1} \geq 0$.

To make this assumption appear “natural” consider the following environment: The short-run investment is risk-free, i.e., it is simply a storage technology. (Consequently, all idiosyncratic or aggregate uncertainty stems from the performance of long-run investments.) A short-run investment is then simply equivalent to holding cash. If all banks choose to hold at least as much cash as they needed in order honor their obligations in $T = 1$,
this implies that Assumption 1 is satisfied. Alternatively, regulatory measures such as reserve requirements may force banks to hold sufficient liquidity. In this case, we may think of Assumption 1 not as resulting from a choice, but as a constraint on the behavior of banks.

**Proposition 3** Suppose that Assumption 1 holds and fix some arbitrary state \( \sigma \) of the economy. Suppose that \( q_0 \) and \( p_0 \) are equilibrium prices for buyers ans sellers, respectively, at a tax rate of \( T = 0 \) and denote by \( \hat{I}_0^I \) the set of bankrupt banks in this equilibrium. Then, for any tax rate \( t \neq 0 \), there is an equilibrium so that:

i) The equilibrium price for buyers is given by \( q_0 \).

ii) The equilibrium price for sellers is \( \frac{q_0}{1 + t} \).

iii) The set of bankrupt banks equals \( \hat{I}_0^I \).

The Proposition establishes a neutrality result. An increase of the tax rate does neither affect the price that buyers have to pay on the financial market nor the number of banks that go bankrupt. It only affects the revenue of a seller. Since the price that buyers pay is not affected by taxation, the price that sellers receive must go down if the tax rate goes up. The intuition is as follows: Assumption 1 eliminates fire-sales. Therefore banks that face a risk of bankruptcy are those with \( \theta_2 < 0 \). These banks have to buy assets on the financial market. Otherwise they will not be able to honor their promises to debtors in \( T = 2 \). Hence, whether these banks survive depends only on the price that buyers face. The number of bankrupt banks and the supply on the financial market therefore become functions of \( q \) only. They no longer depend on the tax rate as an independent argument. (Graphically this means that an increase of the tax rate no longer shifts the supply curve to the right. The supply curve stays where it is.) As a consequence, the equilibrium condition for the buyers’ price, \( S(q) = D(q) \), becomes independent of the tax rate.

### 4.1 Profits and Liquidation Value

In \( T = 0 \), bank \( i \) chooses its investments \( y^0_{i1} \), and \( y^0_{i2} \) and the promises to debtors. Let \( h_{0i} = (y^0_{i1}, y^0_{i2}, x_{i1}, x_{i2}) \) be the vector of variables which are determined in the initial period. Denote by \( q(\sigma, t) \) and \( p(\sigma, t) \) the equilibrium prices for buyers and seller’s respectively, which depend both on the state of the economy \( \sigma \) and the tax rate \( t \). We can now write bank \( i \)'s expected profit as

\[
\Pi^2_i(h_{0i}, t) = E[\sigma][E[\pi^2_i \mid \sigma, p(\sigma, t), q(\sigma, t), h_{0i}]] ,
\]

---

13Such a decision may be motivated by the hope to reduce the risk of bankruptcy or by the hope to have cash reserves that make it possible to reap a profit if the financial market price \( q \) falls below its “fair value” of \( r_2 \).
where the inner expectation is taken conditional on the state $\sigma$ of the economy, a given tax rate of $t$, and on and financial market prices being fixed at levels of $p(\sigma, t)$ and $q(\sigma, t)$, and the bank’s choices in $T = 0$, respectively, and the outer expectation is taken with respect to the random variable $\sigma$.

**Proposition 4** Under Assumption 1, a bank’s expected profit is unaffected by the tax rate $t$. Formally, for any pair of tax rates $t$ and $t'$ with $t \neq t'$ we have that

$$\Pi_2^t(h_{0i}, t) = \Pi_2^t(h_{0i}, t') .$$

The Proposition is a consequence of limited liability. Under Assumption 1 whether or not a bank survives depends on the price $q$ for buyers on the financial market. If it does not survive, limited liability implies that the profit is zero. If it does survive, the bank will be a buyer on the financial market and its profit therefore depends on $q$. By Proposition 3 the price $q$ for buyers on the financial market does not depend on the tax rate. Consequently, a bank’s profit also does not depend on the tax rate $t$. Hence, for given choices in the initial period, $h_{0i}$, an increase of the tax rate only affects the payoff for the debtors who, in case of bankruptcy, receive the liquidation value $y_{i1} + p(\sigma, t)y_{i2}$ of the bank’s asset. By Proposition 3, $p(\sigma, t)$ goes down if the tax rate goes up, which implies that the debtors get less.

**4.2 Investment decisions, Maturity structure of debt**

We now turn to the bank’s initial choices. The bank chooses $h_{0i}$ in order to maximize $\Pi_2^t(h_{0i}, t)$ subject to a participation constraint for its debtors.

We assume that the bank’s debtors are risk averse and, for simplicity, that they are indifferent regarding the timing of the bank’s payment. Hence, if bank $i$ does not go bankrupt the debtors’ utility equals $u(x_{i1} + x_{i2})$, where $u$ is a strictly concave utility function. If bank $i$ does go bankrupt, then the debtors receive the liquidation value of the bank’s asset, and possibly, some government support $b_i$, a bailout or a payment from an insurance system for financial institutions. In this case their utility equals $u(y_{i1} + py_{i2} + b_i)$. We view the bailout as a function of the bank’s initial choices $h_{0i}$, and the state of the economy $\sigma$, and write $b_i = b_i(h_{0i}, \sigma)$. We focus on the case of a complete bailout policy.

**Assumption 2** For all $h_{0i}$ and for all states $\sigma$ in which bank $i$ goes bankrupt,

$$b_i(h_{0i}, \sigma) = x_{i1} + x_{i2} - y_{i1} - p(\sigma)y_{i2} .$$

Under Assumption 2, a debtor’s expected utility can be written as

$$U_i(h_{0i}) = u(x_{i1} + x_{i2}) .$$
The full bailout assumption implies that a debt-holders expected utility does neither depend on the state of the economy, nor on the fortune of the financial institution.

Assumption 2 can be justified in a couple of ways. For commercial banks the assumption is descriptive because of deposit insurance. For investment banks, the recent experience suggests that a full bailout assumption seems like a reasonable approximation. Finally, we have assumed that debtors’ are risk-averse. If we think of the government as a potential provider of insurance to debtors and assume, moreover, that the government is not as risk-averse as debtors, then full bailouts seem to be an optimal arrangement.

Bank \(i\)'s problem now is to choose \(h_{0i} = (y_{i1}^0, y_{i2}^0, x_{i1}, x_{i2})\) in order to maximize \(\Pi_i^2(h_{0i}, t)\) subject to the participation constraint for debtors,

\[
u(x_{i1} + x_{i2}) \geq u_i
\]

where \(u_i\) is a minimal utility level that debtors request in exchange for their willingness to lend to the bank, and the constraint that a bank’s total investments is bounded by the funds that are made available by its depositors and its holders of equity

\[
y_{i1}^0 + y_{i2}^0 = a_i
\]

We denote by \(H_{0i}^*(t)\) the set of solutions to this optimization problem.

**Proposition 5** Under Assumptions 1 and 2, we have that

\[
H_{0i}^*(t) = H_{0i}^*(t'),
\]

for any pair of tax rates \(t\) and \(t'\).

The Proposition follows immediately from two observations: (i) As shown in Proposition 4, the expected profit that is generated by any initial decision \(h_{0i}\) does not depend on the tax rate, and (ii) under a full bailout assumption, also the bank’s constraints do not depend on the tax rate. Consequently, the set of optimal choices for bank \(i\) does not depend on the tax rate.

**Summary: Implications of a FTT under Assumptions 1 and 2**

The analysis so far has shown that if all banks have sufficient liquidity to honor their short-term promises and if debtors are always bailed out if a bank gets in trouble, then the introduction of a financial transactions tax has no relevant economic impact. It does neither affect how many banks get into trouble, nor the prices buyers have to pay on the

\[14\text{The failure of Lehman brothers is the prominent exception where debtors were not made whole. But given the drastic consequences that have been attributed to this political decision, it seems unlikely that this will happen again any time soon.}\]
financial market, nor the bank’s expected profits or a bank’s decisions on its investment portfolio or the optimal maturity structure of debt.

The tax reduces the liquidation value for debtors in case a bank fails and generates some tax revenue for the government. However, if there is always a full bailout by the government, then the tax induced reduction of the liquidation value has to be compensated for by an increase of the payments to debtors. Hence, the tax has no effect on the government’s net revenue, i.e. on the difference between revenues generated by the tax and expenditures due to bailout guarantee.

Given these observations, it does not really matter whether or not a financial transactions tax is introduced. It is neither doing good nor is it doing harm. In the next section we will eliminate Assumptions 1 so that there are indeed banks with liquidity problems. We will see that this changes the analysis in a drastic way.

5 Fire sales

We now study a particular version of the general environment introduced in the previous section. We impose the following assumptions:

**Assumption 3** In $T = 0$, all banks are identical. This implies in particular that they all have the same structure of liabilities, i.e. there exist numbers $X_1$ and $X_2$ so that $x_{i1} = X_1$ and $x_{i2} = X_2$, for all $i$. Also they choose the same short-run and long-run investments in $T = 0$. We denote the common investment levels by $Y^0_1$ and $Y^0_2$, respectively.

Assumption 1 restricts attention to a symmetric environment. While banks may be heterogeneous ex post – that is, after the shock has hit in $T = 1$ – they are identical ex ante.

**Assumption 4** There is aggregate risk with respect to the performance of long run investments. All banks are equally affected by this aggregate risk. Formally, we assume that if bank $i$ invests in $T = 0 y^0_{i2}$ into long-term projects, the return will be equal to $y_{i2} = \sigma y^0_{i2}$, where $\sigma \in [0,1]$ is an aggregate shock that affects all banks equally.

All banks engage in the same long-run investment in $T = 2$ and they will all experience the same outcome, which can be good (high values $\sigma$) or bad (low values of $\sigma$). As an example, suppose that all banks have lend long-term to a pool of home-owners. Within this pool, there is a certain rate of defaults. Since all banks have lent to the same pool, the default rate is the same for all banks. Put differently, their long-term lending risks are perfectly correlated. In a symmetric equilibrium where all banks choose the same
investments – so that there is a number $Y_2^0$ such that $y_{i2}^0 = Y_2^0$, for all $i$—this implies, in particular, that they will experience the same performance of their long-run investments in $T = 1$. Hence, there exists a number $Y_2 = \sigma Y_2^0$ so that $y_{i2} = Y_2$, for all $i$.

**Assumption 5** We assume that $r_2 Y_2 > X_2$, or, equivalently, that $\theta_2 := r_2 Y_2 - X_2 \geq 0$, with probability 1. We also assume that $X_1 > \theta_2$ with probability 1.

The assumption that $\theta_2 := Y_2 - X_2 > 0$ with probability 1 shifts the focus on fire-sales. (In states with $\theta_2 < 0$ there would be no fire-sellers. Instead there would be fire-buyers.) If banks have a lot of short-term debt and only little long-term debt, this means that $X_1$ will be large relative to $X_2$. This situation is typical for investment banks who obtain a lot of their financing on the repo or commercial paper market. A stylized model of this could be to set $X_2 = 0$. In this case, the assumption that $\theta_2 \geq 0$, with probability 1, would follow naturally from the fact that $Y_2$ cannot become negative.

The assumption that $X_1 > \theta_2$ with probability 1, is made for ease of exposition. Economically, this assumption says that short-run liabilities are large relative to the net endowment $\theta_2$ in period 2. As will become clear below, this assumption implies that even at a price of $p = r_2$, i.e. with the most favorable conditions for sellers on the financial market, a bank that experiences a complete failure of its short-run investments, so that $y_{i1} = 0$, will be unable to avoid bankruptcy. Put differently, the assumption ensures that for all $p \in [0, r_2]$, $I_{sell}^\dagger(p) \neq \emptyset$.

**Assumption 6** There is idiosyncratic liquidity risk: Bank $i$'s performing short-run investments are given by $y_{i1} = \beta_i Y_1^0$, where $\beta_i \in [0, 1]$. From the perspective of $T = 0$, $\beta_i$ is a random variable that has a uniform distribution, and is stochastically independent from $Y_2$. Also, with an appeal to the law of large numbers, we assume that the cross-section distribution of $\beta_i$ is a uniform distribution with probability 1, so that there is no aggregate liquidity risk. Finally, we assume that $Y_1^0 > X_1$.

According to the third assumption, the short-run investments gives a random return of $\beta_i$ per unit invested, where the random variable $\beta_i$ captures an idiosyncratic investment risk. The expected return of bank $i$ is $E[\beta_i] = \frac{1}{2}$. Due to the law of large numbers this is also the average return among all banks. The idiosyncratic liquidity risk is what generates a motive for trade on the financial market. Some banks lose a lot on their short-run investments and therefore have to sell some of their assets in order to honor

\footnote{Without this assumption, we would have to distinguish explicitly between low prices such that $I^\dagger_{sell}(p) \neq \emptyset$ and high prices with $I^\dagger_{sell}(p) = \emptyset$. This would be only a minor complication. The supply function derived formally in Proposition 6 would look somewhat different, but it would still be a downward-sloping function.}
their promises and to avoid bankruptcy. Other banks lose very little on their short-run investments and therefore have money left that they can invest on the financial market. The assumption that $Y_1^0 > X_1$ ensures that there are some banks with more money than they need in $T = 1$. Without this assumption there would be no demand on the financial market.

**Proposition 6** Under Assumptions 3 - 6 the demand and the supply functions are, respectively, given by

\[
D(q) = \frac{1}{q} \frac{(Y_1^0 - X_1)^2}{2Y_1^0} \quad \text{and} \quad S(q) = \frac{Y_2}{Y_1^0} X_1 - \frac{\theta_2}{2r_2 Y_1^0} \left( Y_2 + \frac{X_2}{r_2} \right) \frac{q}{1+t}.
\]

The assumption that $\theta_2 > 0$ implies that the supply curve is linear and downward sloping, as in Figures 2 and 3. The demand function $D$ is downward sloping and convex, again as in Figure 2 and 3. This implies that the excess demand function $Z(q) := D(q) - S(q)$ is also globally convex so that there are at most two solutions to the equation $Z(q) = 0$. This implies that, generically, the set of equilibria will have one of the following structures:

(a) There is only one equilibrium with $q = r_2$. This is the case if there is no $q \in [0, r_2]$ with $Z(q) = 0$.

(b) There is one regular equilibrium with $q < r_2$. This is the case if there is exactly one $q \in [0, r_2]$ with $Z(q) = 0$. This situation arises in Figure 3 with the supply function $S_1$.

(c) There is a both a regular and an irregular equilibrium with $q < r_2$. In addition, there is an equilibrium with $q = r_2$. This is the case if there are $q, q' \in [0, r_2]$ with $Z(q) = 0$ and $Z(q') = 0$. This situation arises in Figure 2 and in Figure 3 with the supply function $S$.

In case there are multiple equilibria as in (c) we may again dismiss the irregular equilibrium as implausible, in which case we are left with a “low-price equilibrium” with $q > r_2$ and an equilibrium with $q = r_2$. The latter equilibrium is as if there was unconstrained liquidity so that asset prices are “fair” in the sense that the opportunity cost of buying an asset in $T = 1$, $q$, is equal to the return of $r_2$ that the asset generates in $T = 2$. In the “low-price equilibrium” by contrast, the buyers get the assets at a fire-sales price, i.e. at a price below the fair price. The “low-price equilibrium” and the “fair-price equilibrium” cannot be Pareto-ranked, since the buyers prefer low prices and the sellers prefer high prices. However, the equilibria can be ranked if we view “financial stability” as desirable and agree to measure financial stability by the number of financial institutions in distress. Assumptions 3 - 6 imply that there are no fire-buying banks. Consequently, there is an
unambiguous relation between the financial market price and the number of bankruptcies. The measure of bankrupt banks is given by

\[ \int_{i \in I^*} \left( \frac{q}{r_2^*} \right) di = \int_0^{X_1 - \frac{q}{1 + t} Y_0^*} ds = \frac{1}{Y_1^*} \left( X_1 - \frac{q}{1 + t} Y_0^* \right), \]

which is a strictly decreasing function of \( q \). Hence, from the viewpoint of financial stability – or from the perspective of a government that has to bail out debtors in case of bankruptcy – the “fair-price equilibrium” is also the “good equilibrium” and the “low-price equilibrium” is the “bad equilibrium.”

Assumptions 3 - 6 make it possible to obtain additional comparative statics results on the effects of a small increase of the tax \( t \). Proposition 2 offered a definite answer only with respect to the change of the price \( p \) that sellers receive, but not with respect to the price \( q \) that buyers have to pay after a tax increase. Now, if we focus on a “low-price equilibrium” and consider a small tax increase (graphically, a small rightward shift of the supply curve) this will lead to a decrease of the equilibrium value of \( q \). If, by contrast, we focus on a “fair-price equilibrium”, then we have \( q = r_2^* \) before and after the tax change. These observations are summarized in the following Proposition, which is a strengthened version of Proposition 2 and which we state without proof.

**Proposition 7** Suppose that Assumptions 3 - 6 hold and that there is an initial tax rate \( t_0 \). Consider an equilibrium that is not irregular, i.e. it is either such that \( D(q) = S(q) \) and \( D' < S' \), or it is such that the equilibrium price equals \( r_2^* \) and \( D(r_2^*) > S(r_2^*) \). There exists \( \epsilon > 0 \) so that moving to a higher tax rate \( t_1 \in (t_0, t_0 + \epsilon) \) gives rise to a new equilibrium which is in the neighborhood of the old equilibrium and has the following properties:

i) The new equilibrium price \( p \) that sellers receive is lower.

ii) The new equilibrium price \( q \) that buyers pay is not higher.

iii) The new equilibrium has more bankrupt banks.

Proposition 7 deals with the consequences of a small tax increase that leads to small price adjustments and which leaves the structure of the set of equilibria unchanged. A large increase of the tax rate may however change the set of equilibria. Suppose we are initially in a situation with three equilibria, the bad “low-price equilibrium”, the irrelevant “irregular equilibrium” and the good “fair-price equilibrium”. (Figure 5 provides a numerical example of this situation: With a tax rate of \( t = 0 \) the relevant supply curve is the green one which has two intersections with the red demand curve.) An increase of the tax rate leaves the intercept of the supply curve unaffected but makes it flatter. Hence, if the tax rate is chosen sufficiently high only the “low-price equilibrium” will be left. (In Figure 5, the blue supply curve is relevant for a tax rate of \( t = 0.2 \). It intersects the red
Figure 5: A Numerical example. The parameters choices are $\theta_2 = X_1 = X_2 = Y_2 = 1$, $r_2 = 2$ and $Y_1^0 = 2.1$. The red curve is the demand curve. The green curve is the supply curve for a tax rate of $t = 0$. The blue curve is the supply curve for a tax rate of $t = 0.2$.

demand curve only once.) Alternatively, the initial situation may be one in which the “fair-price equilibrium” is the only equilibrium because the supply curve and the demand curve do not intersect. A substantial increase of the tax rate may then either lead to a situation where a “low-price-equilibrium” and a “fair-price equilibrium” coexist or to a situation where the “low-price-equilibrium” is the only equilibrium, see Figure 6. These observations are summarized in the following Proposition.

Proposition 8 Suppose that Assumptions 3 - 6 hold and that

$$r_2 > \frac{(Y_1^0 - X_1)^2}{Y_2 X_1}.$$ (5)

Suppose that there is an initial tax rate $t_0$ and that, at this tax rate, there exists a “fair-price equilibrium”. There exists $\epsilon > 0$ so that moving to a higher tax rate $t_1 \geq t_0 + \epsilon$ gives rise to a new equilibrium set that contains only a “low-price equilibrium”.

Proposition 8 shows that the introduction of a financial transactions tax can have drastic consequences for financial stability. Suppose that we are in an initial situation in which assets are fairly priced and the number of bankruptcies is modest. Then, an increase of the financial transactions tax may kill the “fair-price equilibrium” and so that only the “low-price equilibrium” is left. Consequently, there would be a large increase in the number of bankruptcies.

5.1 Interaction with other policy measures

The experience of financial instability after the collapse of Lehman brothers in September 2008 has led to a number of policy measures which aimed at restoring confidence on financial markets. These policy measures included liquidity support by central banks
Figure 6: A Numerical example. The parameters choices are $\theta_2 = X_1 = X_2 = Y_2 = 1$, $r_2 = 2$ and $Y_1^0 = 2.5$. The red curve is the demand curve. The green curve is the supply curve for a tax rate of $t = 0$. The blue curve is the supply curve for a tax rate of $t = 0.35$.

(e.g. providing access to the Fed’s discount window to investment banks in the US) and measures (such as the installation of bad banks or the toxic asset relief program in the US) with the intention to stop the vicious spiral of asset price falls which trigger fire-sales which trigger further reductions in asset prices etc. In addition, the excessive reliance of banks on short term funding via the commercial paper or the repo market, has been identified as a major source of financial fragility. This problem could possibly be addressed via cash or solvency requirements that force banks to make sure that they have enough liquidity to honor their short-term promises to debtors. In the following we will discuss these policy measures in the context of our model and study their interaction with a financial transactions tax. Our main finding is that these measure are all effective in the sense that they (i) make it more likely that there is a “fair-price equilibrium” and (ii) reduce the number of bankruptcies in a “low-price equilibrium”. Hence, these measure have good implications from the perspective of financial stability. However, recall from or previous results that the financial transactions tax has exactly the opposite effect: It makes the “fair-price equilibrium” less likely and increases the the number of bankruptcies in a “low-price equilibrium”. Hence, the introduction of a financial transactions tax is problematic in that it neutralizes the beneficial effects of other policy measures that serve to stabilize financial markets.

**Liquidity Support.** We formalize liquidity support by a central bank as a perfectly elastic supply of funds to non-bankrupt banks at an interest rate of 0. The restriction of lending to non-bankrupt banks ensures that the latter will indeed be able to pay back in $T = 2$ what they have lent in $T = 1$.

**Assumption 7** Suppose that all non-bankrupt banks can lend in $T = 1$ as much as they want at an interest rate of 0.
Proposition 9 Under Assumptions 3-7, the “fair-price equilibrium” is the only equilibrium. An increase of the financial transactions tax

i) does not affect the equilibrium price $q$ that buyers have to pay.

ii) depresses the equilibrium price $p$ that sellers receive and therefore increases the number of bankrupt banks.

The fact that banks can lend as much as they want kills the “low-price equilibrium”, which is the bad equilibrium from a financial stability viewpoint. The reason is that, at a price of $q < r_2$, buyers on the financial market get the assets on the financial market below their fair price and make a profit. If there is liquidity support by a central bank, this will trigger a huge demand for liquidity which is then used to buy assets on the financial market. Consequently, at any price $q < r_2$ the demand on the financial market exceeds the supply and this pushes prices up to the equilibrium level $q = r_2$. The beneficial effect of the liquidity support is counteracted by the financial transactions tax. If the price for buyers is fixed at $q = r_2$, then we have that

\[ p = \frac{r_2}{1 + t}, \]

which implies that the price for sellers goes down if the tax rate $t$ goes up, and, consequently, more fire-selling banks go bankrupt.

**TARP/ Bad banks.** An alternative way of restoring financial stability is to limit the amount of assets that are sold on the financial market. In 2008 the US government bought assets from distressed financial institutions via the the toxic asset relief program (**TARP**). An alternative measure is the installation of **bad banks**, i.e. of banks that hold rather than sell assets and which are backed by a government guarantee. The main rationale for these policy measures was to limit the extent of adverse selection problems, by taking the worst, or most-difficult-to value assets from the financial market. In the given model, we have abstracted from issues of adverse selection since we assumed that (i) it becomes commonly known in $T = 1$ which long-term investments fail and which ones perform, and (ii) that all performing long-term investments yield the same return in $T = 2$. We will show in the following that such programs can play a useful role even in the given framework with no adverse selection problem, simply because they limit the volume of fire-sales in a period of financial instability. We model the implications of a **TARP** or **Bad bank**-program via the following assumption.

**Assumption 8** The assets of bankrupt banks are not sold on the financial market.
The financial market price still determines how many banks go bankrupt. The measure of bankrupt banks is given by the formula

$$\int_{i \in I} \frac{q}{(1+t)r} \, di = \int_0^{X_1 - \frac{q}{(1+t)r} \theta_2} \frac{1}{Y_1} \, ds = \frac{1}{Y_1} \left( X_1 - \frac{q}{(1+t)r} \theta_2 \right),$$

However, the price that clears the financial market equates the demand for assets with the supply of assets by fire-selling banks who manage to survive. Without the government intervention, the supply would be larger since also the assets of bankrupt banks would be sold on the financial market. The equilibrium price $q$ is therefore higher and the number of bankruptcies smaller than otherwise.

**Proposition 10** Under Assumptions 3-6 and 8 a financial market equilibrium has the following properties:

i) The supply curve is increasing implying that the equilibrium is unique.

ii) If the unique equilibrium is a “low-price equilibrium”, i.e. an equilibrium with $q < r_2$, then the equilibrium price is higher and the number of bankruptcies is smaller than in a “low price equilibrium” that exists if bankrupt banks sell their assets on the financial market.

iii) An increase of the tax rate leads to a lower price $p$ for sellers and an increased number of bankruptcies. The price $q$ for buyers either goes up or remains constant.

Under Assumption 8 the supply function is increasing, rather than decreasing. A higher price has two effects. It increases the number of banks that can avoid bankruptcy which tends to increase the supply on the market. It also decreases the number of assets that each surviving bank has to sell which tends to decrease the supply. Proposition 10 asserts that the first effect dominates so that the financial market supply becomes an increasing function of the financial market price.

An implication of this is that there is a unique financial market equilibrium. To the extent that equilibrium multiplicity may be considered a problematic source of uncertainty for market participants (e.g. because it implies that is difficult to determine the “true” value of an asset) it is a beneficial side effect of a TARP- or Bad Bank-program that this multiplicity is eliminated.

The implications of a tax increase for the equilibrium value of $p$ and the number of bankruptcies are the by now familiar ones: the price falls and the bankruptcies rise. Since the supply curve is now increasing, the implications of a tax increase for the buyers’ price $q$ are different from those in Proposition 7: $q$ does not fall in response to a tax increase.

Figures 7 and 8 illustrate the implications of TARP- or Bad Bank-program graphically, and also what happens if under such a program the tax rate is increased. In Figure 7,
there is a unique “low-price equilibrium” with and without the program, i.e. with the blue and the green supply curve, respectively. The program gives rise to higher values of \( q \) and \( p \), and hence fewer bankruptcies in case there is no taxation \( t = 0 \). If a tax is introduced the supply curve shifts to the right, i.e. the orange curve is the relevant one. This implies that \( q \) goes up, \( p \) goes down and the number of bankruptcies rises even further. In Figure 8, there are multiple equilibria without the program. With the program only the “fair-price equilibrium” remains. Again, if a tax is introduced the supply curve shifts to the right, implying that \( q \) stays put at a level of \( r_2 \), \( p \) goes down and the number of bankruptcies goes up.

**Equity requirements.** The rationale for equity requirements is that they provide a buffer against losses; that is, if a significant fraction of a bank’s investments fail then, if there is sufficient equity, holders of equity will carry the losses, while the bank will be able to honor the promises to its depositors and therefore avoid bankruptcy.

In the given model, we can think of an equity requirement as follows: We take the amount of debt \( d_i \) and the accompanying claims \( (x_{i1}, x_{i2}) \) of debtors as given. We then study the consequences of an increase of \( e_i \) for the financial market equilibrium. Since \( a_i = e_i + d_i \), this means that, in \( T = 0 \), a bank has additional funds that can be invested either with a short-term perspective or a long-term perspective.

The point we want to make is the following: An increase of equity per se does not yet enhance financial stability. The impact of an equity requirement will depend on how these additional funds are invested. The following Proposition shows that if additional funds are invested with a long-term perspective then financial stability may go down (financial market prices fall and bankruptcies rise), while if they are invested with a short-term
Figure 8: A Numerical example. The parameters choices are $\theta_2 = X_1 = X_2 = Y_2 = 1$, $r_2 = 2$ and $Y_1^0 = 2.1$. The red curve is the demand curve. The green curve is the supply curve for a tax rate of $t = 0$ if failed banks’ assets are sold on the financial market. The blue curve is the supply curve for a tax rate of $t = 0$ if these assets are not sold. The orange curve is the supply curve if these assets are not sold and there is a tax of $t = 0.2$.

perspective financial stability improves. For ease of exposition, we focus on a situation where banks have only short-term debt so that $X_2 = 0$.\footnote{The Proposition trivially extends to situations where $X_2$ is not too large.}

**Assumption 9** Banks have only short-term debt, so that $X_1 > 0$ and $X_2 = 0$.

**Proposition 11** Suppose Assumptions 3-6 and Assumption 9 hold. Suppose there is a “low-price-equilibrium” with an equilibrium price $q^\ast$. Then

i) The equilibrium price $q^\ast$ is an increasing function of $Y_1^0$.

ii) The equilibrium price $q^\ast$ is a decreasing function of $Y_2$.

If additional equity is used for long-run investments – so that $Y_2^0$ goes up – this means that for every state $\sigma$ of the economy $Y_2 = \sigma Y_2^0$ is larger. The Proposition shows that, for all states that give rise to a “low-price equilibrium”, this gives rise to an decrease of the financial market price and therefore to an increased number of bankruptcies. By contrast, if the additional equity is used for short-run investments, so that $Y_1^0$ increases, this leads to an increase of the financial market price and therefore to a reduced number of bankruptcies.

The intuition for this observation is as follows: Since $X_2 = 0$, all banks trivially have sufficient assets to honor their obligations in $T = 2$. Hence, the only source of financial distress is that banks may have insufficient liquidity in $T = 1$. This problem is not alleviated if the amount invested with a long-run perspective goes up. The problem is even aggravated since those banks that fail throw more assets on the financial market
and thereby generate downward pressure on the market price. By contrast, additional investments with a short-run perspective have a beneficial effect. At an aggregate level, they increase the liquidity that is available in $T = 1$. This increase of aggregate liquidity implies that the demand on the financial market is higher and this leads to a upward pressure on the financial market price.

The important insight is that an equity requirement does not by itself contribute to financial stability. To have a beneficial effect, additional equity must be invested in such a way that the proceeds from the investment are available in the short-run. Again, as an implication of Proposition 7, a beneficial use of equity which drives the financial market price up, will be neutralized by the introduction of a financial transactions tax which drives the price $p$ that seller’s receive down.

5.2 Investment decisions, Maturity structure of debt

We now turn to a bank’s choice of $h_{0i} = (y_{0i1}, y_{0i2}, x_{i1}, x_{i2})$ in the initial period. The following Proposition gives a formula for bank $i$’s expected profits.

**Proposition 12** Under Assumptions 3-6,

\[
\Pi_{i}^{2}(h_{0i}, t) = \frac{1}{2} c_{1} \frac{y_{11}^{0}}{y_{11}^{0}} E_{\sigma} \left[ \frac{r_{2}}{q(\sigma, t)} (y_{11}^{0} - x_{i1})^{2} + 2(y_{11}^{0} - x_{i1})(r_{2} \sigma y_{12}^{0} - x_{i2}) + \frac{p(\sigma, t)}{r_{2}} (r_{2} \sigma y_{12}^{0} - x_{i2})^{2} \right] \tag{6}
\]

We again impose Assumption 2 so that debtors are always made whole by the government in case bank $i$ goes bankrupt. Bank $i$’s problem therefore is to choose $h_{0i}$ in order to maximize $\Pi_{i}^{2}(h_{0i}, t)$ subject to the constraints that $u(x_{i1} + x_{i2}) = \bar{u}$ and $y_{11}^{0} + y_{12}^{0} = a_{i}$. Assume, for ease of notation, that non-negativity constraints can be neglected. Then, a solution

\[
h_{01}^{*}(t) = \left((y_{011}^{*}(t), y_{012}^{*}(t), x_{11}^{*}(t), x_{12}^{*}(t))\right) \tag{7}
\]

satisfies the following first order conditions

\[
\frac{\partial \Pi_{i}^{2}(h_{0i}^{*}(t), t)}{\partial y_{11}^{0}} = \frac{\partial \Pi_{i}^{2}(h_{0i}^{*}(t), t)}{\partial y_{12}^{0}}, \tag{8}
\]

and

\[
\frac{\partial \Pi_{i}^{2}(h_{0i}^{*}(t), t)}{\partial x_{11}} = \frac{\partial \Pi_{i}^{2}(h_{0i}^{*}(t), t)}{\partial x_{12}}, \tag{9}
\]

which require, respectively, that the marginal returns of long- and short-term investments are equalized and that the marginal costs (in terms of forgone profit) of long and short-term debt are equalized. The following Proposition shows how the solution to this system of equations changes if the tax rate changes.
Proposition 13 Suppose that, for all $\sigma$, $q(\sigma, t)$ and $p(\sigma, t)$ are regular equilibrium prices and differentiable functions of the tax rate $t$. Also suppose that, for all $i$, $y^0_{i1} > x_{i1}$ and $\theta^{i2} := r_2 \sigma y^0_{i2} - x_{i2} > 0$, with probability 1, and that bank $i$’s optimal decision in $T = 0$ is characterized by the first order conditions in (7) and (8). Then, bank $i$ responds in the following way to an increase of the tax rate:

i) It increases its short-run investments and decreases its long-run investments.

ii) It decreases its short-run debt and increases its long-run debt.

The Proposition restricts attention to local changes of equilibrium prices in response to taxation. In addition it imposes Assumption 5 for each bank separately, i.e. it is assumed that, for all $i$, $y^0_{i1} > x_{i1}$ and $\theta^{i2} := r_2 \sigma y^0_{i2} - x_{i2} > 0$, with probability 1. Under these assumptions the Proposition shows that there is a beneficial effect of an increase of the financial transaction tax: Banks reduce their maturity mismatch, i.e., they reduce their short-term debt and increase their short-term investments.\textsuperscript{17} The intuition is straightforward: A tax increase makes it less attractive to be a fire-seller. The resulting decline of the price $p$ that sellers get, makes bankruptcy more likely, and decreases the profits of those fire-sellers that manage to survive. Hence, it becomes more attractive to be a safe bank rather than a fire-seller. The chance of being safe is increased if the maturity mismatch is reduced.

For reasons of tractability, the Proposition limits attention to local changes whose impact can be analyzed by looking at derivatives. We have seen previously, however, that tax increases can change the equilibrium structure so that, for some states $\sigma$, there is a jump from a “fair-price equilibrium” to a “low-price equilibrium”. Allowing for such drastic shifts would reinforce the conclusions of Proposition 13. What is essential for the Proposition, is the price response to a tax increase, namely that the price $p$ falls in every state and that the price $q$ either remains constant or falls. Now, the jump from a “fair-price equilibrium” to a “low-price equilibrium” has exactly this structure: both prices fall. Hence, Proposition 13 would extend if this possibility was acknowledged.

To sum up, we have seen that a financial transactions tax has two different, and opposing implications for financial stability. In the short-run, for given investment decisions and debt structures, such a tax threatens financial stability. It depresses the prices on the financial markets and thereby makes life more difficult for a distressed financial institution. In the long run, however, banks will adjust their investments and their debt structure so that maturity mismatch is reduced and distress becomes more likely. At first glance, this raises the question of how to strike a balance between these two effects. However, the beneficial corrective effects of the transactions tax could be generated in a

\textsuperscript{17}Recall from Proposition 11 that this leads to an increase of financial market prices and hence to fewer bankruptcies.
different way, namely by a tax that addresses maturity mismatch directly. This would also generate more short-term investments and more long-term debt without having the destabilizing impact of a tax on financial transactions.

6 Concluding Remarks

This paper has studied the incidence of a financial transactions tax using a particular model of a financial market. In this model, the purpose of financial markets is to facilitate trade between banks with excess liquidity and banks with deficient liquidity. The freeze of interbank markets that was experienced after the collapse of Lehman brothers in September 2008, and the repercussions that this generated for the real economy, suggests that this approach is appropriate if one is interested in the implications of a financial transactions tax for financial stability. The main insight of the paper is that, for such a market, a financial transactions tax has undesirable implications: It generates more financial distress.

References


**Appendix**

**Proof of Proposition 1.** We first show that $D(q) > S\left(\frac{q}{1+t}\right)$ as $q$ approaches 0. Hence, at the minimal price of $q = p = 0$, demand exceeds supply on the financial market. Note that, for any $q$, $S\left(\frac{q}{1+t}\right)$ is bounded from above. This follows from the assumptions that in $T = 0$, each bank has a limited amount of funds that can be invested in the long asset, and that the number of performing assets $T = 1$ cannot not exceed the investments made in $T = 0$. From

$$D(q) = \int_{I^*(q)} \frac{\theta_{i_1}}{q} di$$

and the assumption that $\hat{I}^*(0) \neq \emptyset$ it follows that

$$\lim_{q \to 0} D(q) = \infty.$$ 

Now, since $D$ and $S$ are assumed to be continuous, there are only two possibilities: Either they do not intersect over the domain $[0, r_2]$. In this case it has to be true that
\(D(r_2) > S\left(\frac{r_2}{1+t}\right)\) so that \(r_2\) is an equilibrium price. Or they do intersect, implying the existence of a price \(q^* < r_2\) so that \(D(q^*) = S\left(\frac{q^*}{1+t}\right)\) in which case \(q^*\) is an equilibrium price.

**Proof of Proposition 2.** For both types of equilibria, i.e. those with \(D(q) = S(q)\) and \(D' < S'\) and those with \(D(r_2) > S(r_2)\), moving to a higher tax rate \(t_1 \in (t_0, t + \epsilon)\) gives rise to a new equilibrium which is in the neighborhood of the old equilibrium. This can be illustrated graphically: The tax increase generates a “small” rightward shift of the supply curve. If the initial equilibrium is one with \(S(q) = D(q)\) and \(D' < S'\) then, because of the continuity of \(D\) and \(S\), the new equilibrium will inherit both of these properties. If, by contrast, the initial equilibrium is such that \(q_0 = r_2\) and \(D(r_2) > S(r_2)\), then, because of the continuity of \(D\) and \(S\), after a small rightward shift of \(S\) it is still the case that \(D(r_2) > S(r_2)\) implying that \(q_1 = r_2\) is still an equilibrium price. With \(p_0 = \frac{q_0}{1+t_0} = \frac{r_2}{1+t_0}\) and \(p_1 = \frac{q_1}{1+t_1} = \frac{r_2}{1+t_1}\), the conclusion \(p_0 < p_1\) then follows immediately for the latter type of equilibrium.

Now, consider an equilibrium with \(D(q) = S(q)\) and \(D'(q) < S'(q)\). We denote by \(q^*(t)\) the equilibrium price for buyers as a function of the tax rate \(t\). This price is implicitly defined by the equation \(D(q^*(t)) = S\left(\frac{q^*(t)}{1+t}\right)\). Analogously we define \(p^*(t) := \frac{q^*(t)}{1+t}\). Straightforward calculations yield

\[
p^*(t) = \frac{q^*(t)}{(1+t)^2} \frac{D'(q^*(t))(1+t)}{S'(\frac{q^*(t)}{1+t}) - D'(q^*(t))(1+t)} \tag{9}
\]

For any \(q\), and any given tax rate \(t\), we have that \(S(q) = S\left(\frac{q}{1+t}\right)\). This implies that

\[
S'(q)(1+t) = S\left(\frac{q}{1+t}\right) \tag{10}.
\]

Using (10), we can rewrite equation (9) as

\[
p^*(t) = \frac{q^*(t)}{(1+t)^2} \frac{D'(q^*(t))}{S'(q^*(t)) - D'(q^*(t))} \tag{11}.
\]

Since \(D' < 0\), \(p^*(t) < 0\) if and only if \(S' > D'\), i.e., if the initial equilibrium is regular.

To complete the proof we note that the claim in ii) is an immediate consequence of the definition of the set \(I_{sell}^*(p)\).

\[
\square
\]

**Proof of Proposition 3. ad i)** Under Assumption 1, there are no failed banks and no fire-selling banks so that \(I^1 = I_{sell}^1(p) = I_{sell}^*(p) = \emptyset\). The set of bankrupt banks consists entirely of fire-buying banks that fail, i.e., of the set \(I_{buy}^1(q)\). Note that whether or not a
bank fails depends only on the price \( q \) for buyers but not on the price \( p \) for sellers. The supply on the financial market is therefore given by

\[
S(q) = \int_{I^{buy}(q)} y_{i2} \, di.
\]

Observe that this supply function depends only on \( q \), whereas in the general model of the previous section it depended on \( q \) and \( t \). Consequently, the equilibrium condition – \( q \) is an equilibrium price if (i) \( D(q) = S(q) \) and \( q \leq r_2 \), or (ii) \( D(q) > S(q) \) and \( q = r_2 \) – is independent of the tax rate. Therefore if \( q_0 \) fulfills the equilibrium condition for a tax rate of 0 then it fulfills the equilibrium condition for any tax rate.

\( \text{ad ii)} \) Immediate from part i) and the condition that \( q = (1 + t)p \).

\( \text{ad iii)} \) Immediate from part i) and the observation in the proof of part i) that the set of bankrupt banks depends only on \( q \).

**Proof of Proposition 4.** Under Assumption 1 there are only three types of banks: safe ones, fire-buying banks that go bankrupt and fire-buying banks that survive. Fix a state of the economy \( \sigma \). Since banks are protected by limited liability, If \( \theta_{i2} < 0 \) and \( \frac{q(\sigma,t)}{r_2} > -\frac{\theta_{i1}}{\theta_{i2}} \), a bank’s profit is zero. Otherwise the bank makes a profit. Profit maximizing behavior of safe and fire-buying banks implies that the profit equals \( \theta_{i2} + \frac{r_2}{q(\sigma,t)} \theta_{i1} \). We can therefore write

\[
E[\pi^2 \mid \sigma, p(\sigma,t), q(\sigma,t), h_0] = E \left[ 1 \left( \theta_{i2} + \frac{r_2}{q(\sigma,t)} \theta_{i1} \geq 0 \right) \left( \theta_{i2} + \frac{r_2}{q(\sigma,t)} \theta_{i1} \right) \mid \sigma, q(\sigma,t), h_0 \right].
\]

Observe that the right hand side of this equation does not depend on \( p(\sigma,t) \). Consequently, the ex ante expected profit

\[
\Pi^2_1(h_0, t) = E_\sigma \left[ E \left[ 1 \left( \theta_{i2} + \frac{r_2}{q(\sigma,t)} \theta_{i1} \geq 0 \right) \left( \theta_{i2} + \frac{r_2}{q(\sigma,t)} \theta_{i1} \right) \mid \sigma, q(\sigma,t), h_0 \right] \right],
\]

also does not depend on \( p(\sigma,t) \). It does depend on \( q(\sigma,t) \). In Proposition 3, however, we have shown that for all \( \sigma, t \) and \( t' \) that \( q(\sigma,t) = q(\sigma,t') \). Consequently, we have for every \( \sigma, t \) and \( t' \) that

\[
E \left[ 1 \left( \theta_{i2} + \frac{r_2}{q(\sigma,t)} \theta_{i1} \geq 0 \right) \left( \theta_{i2} + \frac{r_2}{q(\sigma,t)} \theta_{i1} \right) \mid \sigma, q(\sigma,t), h_0 \right] =
E \left[ 1 \left( \theta_{i2} + \frac{r_2}{q(\sigma,t')} \theta_{i1} \geq 0 \right) \left( \theta_{i2} + \frac{r_2}{q(\sigma,t')} \theta_{i1} \right) \mid \sigma, q(\sigma,t'), h_0 \right]
\]

and therefore also that

\[
\Pi^2_1(h_0, t) = \Pi^2_1(h_0, t').
\]
Proof of Proposition 5. By Proposition 4, under Assumption 1,
\[ \Pi^2_i(h_{0i}, t) = \Pi^2_i(h_{0i}, t') , \]
for any pair of tax rates \( t \) and \( t' \) and any vector \( h_{0i} \). Hence, there exists a function \( \bar{\Pi}^2_i : h_{0i} \mapsto \Pi^2_i(h_{0i}) \) such that
\[ \bar{\Pi}^2_i(h_{0i}) = \Pi^2_i(h_{0i}, t) , \]
for any tax rate \( t \). Under Assumption 2, bank \( i \)'s problem is therefore equivalent to the problem of choosing \( h_{01} \) in order to maximize \( \bar{\Pi}^2_i(h_{0i}) \) subject to the constraints
\[ u(x_{i1} + x_{i2}) \geq \bar{u}_i \quad \text{and} \quad y_{i1}^0 + y_{i2}^0 = a_i . \]
Neither the objective function nor any of the constraints depends on the tax rate \( t \). Hence, if \( h_{0i} \) is an optimal choice for some tax rate, then it is an optimal choice for any tax rate.
\[ \square \]

Proof of Proposition 6. We first derive the demand and the supply function. The assumption that \( \theta_2 > 0 \) implies that \( I^\dagger = I^\dagger_{\text{buy}}(q) = I^\dagger_{\text{buy}}(q) = \emptyset \), for all \( q \). The demand for assets on the financial market is therefore given by
\[ D(q) = \int_{i \in I^*} \frac{\theta_{i1}}{q} \, di . \]
Under Assumptions 3 - 6, this can be written as
\[ D(q) = \int_{X_1}^{Y_0^q} \frac{s - X_1}{q} \frac{1}{Y_0^q} \, ds = \frac{1}{q} \frac{(Y_0^q - X_1)^2}{2Y_0^q} . \]
The supply on the financial market stems from banks in \( I^\dagger_{\text{sell}}(p) \) who go bankrupt and whose assets are liquidated and from banks in \( I^*_{\text{sell}}(p) \) that fire-sell and manage to survive. The supply of the former is given by
\[ \bar{S}^\dagger(p) := \int_{i \in I^\dagger_{\text{sell}}(p)} Y_2 \, di = Y_2 \int_X^{X_1 - \frac{\theta_2}{r_2}} \frac{1}{Y_0^q} \, ds = \frac{Y_2}{Y_1^q} \left( X_1 - \frac{p}{r_2} \theta_2 \right) . \]
The supply of the latter is given by
\[ \bar{S}^*(p) := -\int_{i \in I^*_{\text{sell}}(p)} \frac{\theta_{i1}}{p} \, di = \int_{X_1 - \frac{p}{r_2} \theta_2}^{X_1} \frac{X_1 - s}{p} \frac{1}{Y_0^q} \, ds = \frac{1}{2} \frac{p}{Y_1^q} \left( \frac{\theta_2}{r_2} \right)^2 . \]
Summing these two expressions and exploiting that \( \theta_2 = r_2 Y_2 - X_2 \) yields
\[ \bar{S}(p) = \frac{Y_2}{Y_1^q} X_1 - \frac{\theta_2}{2r_2 Y_1^q} \left( Y_2 + \frac{X_2}{r_2} \right) p \]
and hence
\[ S(q) = \frac{Y_2}{Y_1^q} X_1 - \frac{\theta_2}{2r_2 Y_1^q} \left( Y_2 + \frac{X_2}{r_2} \right) \frac{q}{1 + t} . \]
\[ \square \]
Proof of Proposition 8. The supply curve has an intercept of $\frac{Y_0^*}{Y_2}X_1$. The assumption
\[ r_2 > \frac{(Y_1^0 - X_1)^2}{Y_2X_1}. \]
ensures that a horizontal supply curve with this intercept (the limit case which arises as $t \to \infty$) intersects the demand curve at a price $q < r_2$. To see this, note that the price which equates $D(q)$ and $\frac{Y_2}{Y_1^0}X_1$ is given by
\[ q = \frac{(Y_1^0 - X_1)^2}{Y_2X_1}. \]
Now if (5) is violated then there is no intersection of the supply and the demand curve at a price $q < r_2$, whatever the tax rate. Put differently, the “fair-price equilibrium” would be the only equilibrium whatever the tax rate. By contrast, if (5) holds, then there exists a critical tax rate $\hat{t}$ so that for all $t \geq \hat{t}$, the “low-price equilibrium” is the only equilibrium. \qed

Proof of Proposition 9. Suppose there is an equilibrium with $q < r_2$. Then, if a bank lends $q$ units of money from the central bank it can buy one asset and this asset will return $r_2$ in $T = 2$. In $t_2$ the bank has to repay $q$ and therefore makes a profit of $r_2 - q > 0$. Hence, all banks will lend unlimited amounts from the central bank and there will be an arbitrarily large demand on the financial market. Since the supply of assets on the financial market is bounded this implies that $q < r_2$, and $D(q) > S(q)$. This contradicts the assumption that there is an equilibrium with $q < r_2$. There can also be no equilibrium with $q > r_2$, because in this case all non-bankrupt banks strictly prefer to hold cash over the purchase of assets on the financial market. Consequently, in any equilibrium, it has to be the case that $q = r_2$. This implies that, in equilibrium,
\[ p = \frac{q}{1 + t} = \frac{r_2}{1 + t}, \]
so that an increase of $t$ reduces $p$. The measure of bankrupt banks is given by
\[ \int_{i \in R^*_c((\frac{q}{1+t}))} di = \int_0^{X_1 - \frac{q}{1+(1+r_2)2}} \frac{1}{Y_1^0} ds = \frac{1}{Y_1^0} \left( X_1 - \frac{q}{(1+t)r_2} \theta_2 \right), \]
With $q = r_2$ this simplifies to
\[ \frac{1}{Y_1^0} \left( X_1 - \frac{1}{(1+t)} \theta_2 \right) \]
which is an expression that is strictly increasing in $t$. \qed
Proof of Proposition 10.  

\textit{ad i)} It follows from the argument in the proof of Proposition 6 that, under Assumption 10, the supply curve is given by

\[ S^\ast(q) = \frac{1}{2} \left( \frac{\theta_2}{r_2} q \right)^2. \]

Obviously, this curve is increasing and hence has a unique intersection \( q' \) with the demand curve. If \( q' \leq r_2 \), then \( q' \) is the unique equilibrium price. Otherwise \( r_2 \) is the unique equilibrium price.

\textit{ad ii)} Under Assumption 10 supply is given by \( S^\ast \). Absent the government intervention supply is given by

\[ S(q) = S^\ast(q) + S^\dagger(q), \]

where \( S^\dagger \) is defined formally in the proof of Proposition 6. Under Assumption 5 we have that \( S^\dagger > 0 \), for all \( q < r_2 \). Hence, for all \( q < r_2 \), we have that

\[ S(q) > S^\ast(q) \]

Consequently, if \( q' \) is such that \( S(q') = D(q') \), then \( D(q') > S^\ast(q') \) and \( q'' > q' \) for any \( q'' \) satisfying \( D(q'') = S(q'') \).

\textit{ad iii)} Consider a “low-price equilibrium.” A small tax increase shifts the supply curve to the right and yields a new “low-price equilibrium”. Since \( S' > 0 \) and \( D' < 0 \) the new equilibrium has a higher value of \( q \) and, by Proposition 2, a strictly lower value of \( p \) and therefore a strictly larger number of bankrupt banks. A larger tax increase may eventually turn this “low-price equilibrium” into a “fair-price equilibrium”. However, by the preceding argument, \( p \) falls and bankruptcies rise all along the way to the “fair-price equilibrium”. Once we are in a “fair-price equilibrium”, further tax increases leave the equilibrium value of \( q \) unaffected but depress the equilibrium value of \( p \) further, which leads to more bankruptcies. \( \square \)

Proof of Proposition 11. Under Assumptions 3-6 and Assumption 9 demand and supply are given by

\[ D(q) = \frac{1}{q} \frac{(Y_1^0 - X_1)^2}{2Y_2^2} \quad \text{and} \quad S(q) = \frac{Y_2}{Y_1^2} X_1 - \frac{1}{2Y_1^2} (Y_2)^2 \frac{q}{1+t}. \]

The equation \( D(q) = S(q) \) has two solutions. We focus on a “low-price-equilibrium” that is on the smallest \( q \) such that \( D(q) = S(q) \), henceforth referred to as \( q^\ast \). Straightforward computations yield

\[ q^\ast = \frac{(1 + t)X_1 - \sqrt{(1 + t)^2 (X_1)^2 - (1 + t)(Y_1^0 - X_1)^2}}{Y_2}, \]

an expression which increases in \( Y_1^0 \) and decreases in \( Y_2 \). \( \square \)
Proof of Proposition 12. We can, for any given pair of financial market prices $p$ and $q$, write bank $i$’s expected return on equity as

\[ E[\pi_i^2 | \sigma, p, q, h_{0i}] = \frac{1}{\epsilon_i} \left( V_i^{\text{buy}}(\sigma, q, h_{0i}) + V_i^{\text{sell}}(\sigma, p, h_{0i}) \right), \tag{12} \]

where

\[ V_i^{\text{buy}}(\sigma, q, h_{0i}) := E \left[ 1 \left( \theta_{i1} \geq 0, \frac{q}{\sigma} \theta_{i2} \geq 0 \right) \left( \theta_{i2} + \frac{\sigma}{q} \theta_{i1} \right) | \sigma, q, h_{0i} \right] \]

is the expected return on equity conditional on bank $i$ having excess liquidity in $T = 1$ and therefore being a buyer on the financial market in $T = 1$, and

\[ V_i^{\text{sell}}(\sigma, p, h_{0i}) := E \left[ 1 \left( \theta_{i1} \leq 0, \frac{p}{\sigma} \theta_{i2} \geq 0 \right) \left( \theta_{i2} + \frac{\sigma}{p} \theta_{i1} \right) | \sigma, p, h_{0i} \right] \]

is the expected return on equity conditional on bank $i$ having to fire-sell on the financial market in $T = 1$. In these expressions $1$ is the indicator function and expectations are taken with respect to the random variables $\theta_{i1}$ and $\theta_{i2}$, respectively. Assumptions 5 and 6 imply that

\[ V_i^{\text{buy}}(\sigma, q, h_{0i}) = \int_{x_{i1}}^{y_{i1}} \left( \frac{\sigma}{q} \theta_{i2} + \frac{\sigma}{q} (s - x_{i1}) \right) \frac{1}{y_{i1}} ds = \left( 1 - \frac{x_{i1}}{y_{i1}} \right) \left( \frac{p}{\sigma} \theta_{i2} y_{i1}^0 - x_{i2} + \frac{1}{2} \frac{p}{\sigma} (y_{i1}^0 - x_{i1}) \right). \]

Assumptions 5 and 6 also imply that

\[ V_i^{\text{sell}}(\sigma, q, h_{0i}) = \int_{x_{i1} - \frac{p}{\sigma} \theta_{i2}}^{x_{i1}} \left( \frac{\sigma}{p} \theta_{i2} + \frac{\sigma}{p} (s - x_{i1}) \right) \frac{1}{y_{i1}} ds = \frac{1}{2} \frac{p}{\sigma} \frac{\theta_{i2} y_{i1}^0 - x_{i2}}{y_{i1}^0}. \]

Evaluating these expressions for $V_i^{\text{sell}}(\sigma, q, h_{0i})$ and $V_i^{\text{sell}}(\sigma, p, h_{0i})$ at the equilibrium prices $q(\sigma, t)$ and $p(\sigma, t)$, respectively, and substituting them into (12) yields

\[ E[\pi_i^2 | \sigma, p(\sigma, t), q(\sigma, t), h_{0i}] = \frac{1}{\epsilon_i} \left( \left( 1 - \frac{x_{i1}}{y_{i1}} \right) \left( \frac{p}{\sigma} \theta_{i2} y_{i1}^0 - x_{i2} + \frac{1}{2} \frac{p}{\sigma} (y_{i1}^0 - x_{i1}) \right) + \frac{1}{2} \frac{p(\sigma, t)}{\sigma} \frac{(y_{i1}^0 - x_{i2})^2}{y_{i1}^0} \right). \tag{13} \]

The proposition now follows from the law of iterated expectations which implies that

\[ \Pi_i^2(h_{0i}, t) = E_{\sigma} \left[ E[\pi_i^2] | \sigma, p(\sigma, t), q(\sigma, t), h_{0i} \right]. \]

and some straightforward algebraic manipulations.  \( \square \)
Proof of Proposition 13. Fix some arbitrary vector $h_{01}$. Using equation (6) we can compute the following expressions

$$\frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \partial y_{i1}^0} = \frac{-1}{2 e_i (y_{i1}^0)^2} E_{\sigma} \left[ \frac{r_2}{q(\sigma, t)^2} \frac{\partial q(\sigma, t)}{\partial t} ((y_{i1}^0)^2 - (x_{i1})^2) \right.\
\left. + \frac{\partial p(\sigma, t)}{\partial t} (r_2 \sigma y_{i2}^0 - x_{i2})^2 \right],$$

$$\frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \partial x_{i1}} = \frac{1}{e_i y_{i1}^0} E_{\sigma} \left[ \frac{r_2}{q(\sigma, t)^2} \frac{\partial q(\sigma, t)}{\partial t} (y_{i1}^0 - x_{i1}) \right],$$

$$\frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \partial y_{i2}^0} = \frac{1}{e_i y_{i1}^0} E_{\sigma} \left[ \frac{1}{r_2} \frac{\partial p(\sigma, t)}{\partial t} r_2 \sigma (r_2 \sigma y_{i2}^0 - x_{i2}) \right],$$

and

$$\frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \partial x_{i2}} = \frac{-1}{e_i y_{i1}^0} E_{\sigma} \left[ \frac{1}{r_2} \frac{\partial p(\sigma, t)}{\partial t} (r_2 \sigma y_{i2}^0 - x_{i2}) \right].$$

It follows from Proposition 7 that, for all $\sigma$,

$$\frac{\partial q(\sigma, t)}{\partial t} \leq 0 \quad \text{and} \quad \frac{\partial p(\sigma, t)}{\partial t} < 0.$$

Using these results and the assumption that, $y_{i1}^0 > x_{i1}$ and $\theta_{i2} := r_2 \sigma y_{i2}^0 - x_{i2} > 0$, with probability 1, makes it possible to verify the following statements

$$\frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \partial y_{i1}^0} > 0,$$

$$\frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \partial x_{i1}} \leq 0,$$

$$\frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \partial y_{i2}^0} < 0,$$

$$\frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \partial x_{i2}} > 0.$$

Consequently, if we have an initial situation $h_{0i}$ in which the first order condition in (7) is satisfied, and then increase the tax rate, the first-order condition is violated and we have that

$$\frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \partial y_{i1}^0} > \frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \partial y_{i2}^0},$$

i.e. it becomes optimal to increase $y_{i1}^0$ and to decrease $y_{i2}^0$. Analogously, if initially the first order condition 8 is satisfied, then after a tax increase we have

$$\frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \partial x_{i2}} > \frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \partial x_{i1}},$$

so that it becomes optimal to increase $x_{i2}$ and to decrease $x_{i1}$. □