Abstract: Within a two-country model with involuntary unemployment, this paper investigates corporate income taxation under separate accounting versus formula apportionment. In contrast to separate accounting, under formula apportionment the corporate tax policy causes a fiscal externality that results from unemployment. This externality is the highest when the apportionment formula contains the payroll factor only. It is minimized for the pure sales (property) formula, if the substitution elasticity is low (high). The unemployment externality tends to compensate other externalities such that tax rates become inefficiently low. The deviation from the efficient tax policy is minimized when the apportionment formula contains the sales factor only.

JEL classification: H25, H71, J60

key words: separate accounting, formula apportionment, unemployment
1 Introduction

Corporate income taxation of multinational enterprises (MNEs) can basically follow two principles. Under separate accounting, profit of a MNE is taxed in the country where the MNE it declares. This principle is in operation mainly at the international level. Under formula apportionment, in contrast, corporate income of the MNE’s subsidiaries is first consolidated and then allocated back to the taxing countries according to a formula that usually contains the MNE’s capital (property), sales and payroll shares. Formula apportionment is applied at the national level, for example, in the US, Canada and Germany. Recently, the discussion of the relative merits of the two tax principles received a renewed interest both among politicians and researchers since the European Commission (2007a-c) presented plans to replace separate accounting by formula apportionment within the boarders of the European Union.

This renewed interest is also reflected by an increasing number of economic studies on the two tax principles. The initial study of Gordon and Wilson (1986) has recently been augmented by, for example, Eggert and Schjelderup (2003), Nielsen et al. (2003, 2004), Sørensen (2004), Kind et al. (2005), Gérard (2005, 2006), Pethig and Wagener (2007), Riedel and Runkel (2007) and Pinto (2007). This literature shows that the comparison of the two tax principles is basically a theory of fiscal externalities. Under separate accounting, corporate taxation causes a positive profit shifting externality that points to inefficiently low tax rates. Under formula apportionment, corporate taxation is characterized by a tax base externality and a formula externality. Under mild conditions, the sum of these two externalities is positive and points to inefficiently low tax rates as under separate accounting (e.g. Haufler 2006). Nevertheless, the existence of positive corporate income implies a tax exporting effect à la Huizinga and Nielsen (1997) so that under both tax principles there additionally emerges a negative income externality which may render tax rates inefficiently high.

All above studies either ignore labor as a factor of production or consider perfect labor markets with full employment. Obviously, reality draws a quite different picture. Since the early seventies the member countries of the EU have been suffering from drastically growing unemployment. According to Eurostat data, the average unemployment rate in the EU-25 countries is currently around 9%. Even though unemployment in the US is typically lower than in the EU, also the US faces an unemployment prob-
lem. The current unemployment rate in the US is about 5%.\textsuperscript{1} These observations immediately raise the question how unemployment affects the efficiency properties of corporate income taxation under the two tax principles.

The present paper addresses this question. It investigates corporate income taxation in a model with involuntary unemployment. We develop a two-country model with a representative MNE. In each country, the MNE produces an output with capital and labor as inputs. The MNE may shift profit from one country to the other by, for example, distorting transfer prices. Unemployment is modeled by assuming that due to minimum wage legislations the wage rate is fixed at a level above the equilibrium wage rate so that labor demand falls short of labor supply. Each country is populated by a continuum of households some of which are unemployed due to the labor market imperfection. The countries impose a corporate income tax on the MNE and non-cooperatively choose their tax rates in order to maximize welfare of their residents.

Within this model, we first identify the fiscal externalities caused by corporate income taxation. Under separate accounting, it turns out that taking into account unemployment does not add a further fiscal externality to the above mentioned externalities, since the corporate tax in one country leaves unchanged the MNE’s labor demand in the other country. In contrast, under formula apportionment a cross country effect of corporate taxation on unemployment emerges for two reasons. First, corporate taxes fall on the consolidated tax base so that an increase in one country’s tax rate induces the MNE to reduce labor demand in both countries (tax base effect). Second, as a reaction on a tax increase in one country the MNE reallocates labor to the other country in order to reduce the share of consolidated profit which the apportionment formula assigns to the tax increasing country (formula effect). The sum of both effects turn out to be positive and, thus, constitutes a positive unemployment externality which so far has been ignored in the literature and which points to inefficient undertaxation. As intuitive plausible, we can show that the unemployment externality is the highest when the apportionment formula contains the payroll factor only. It is minimized for the pure sales (property) formula, if the substitution elasticity is low (high).

We also investigate the interplay of the unemployment externality and the above mentioned externalities known from previous studies. We show that the sum of tax

\textsuperscript{1} All these data are taken from the Eurostat website under http://epp.eurostat.cec.eu.int.
base and formula externalities is positive. Moreover, if the production elasticity of labor is sufficiently large, the unemployment externality will be strong enough to outweigh the income externality such that the sum of both is positive, too. Under formula apportionment, the corporate income tax rates are then inefficiently low. This result is to a large extent driven by the newly derived unemployment externality since it compensates the income externality, i.e. that externality which may be responsible for too high tax rates if unemployment is ignored. In addition, starting from the assumption that tax rates under formula apportionment are inefficiently low, we show that under certain conditions the efficiency gap will be minimized if the apportionment formula exclusively uses a sales factor.

To the best of our knowledge, the only study investigating formula apportionment in the presence of unemployment is that of Goolsbee and Maydew (2000). In contrast to our analysis, however, they do not theoretically analyze the implications of unemployment under formula apportionment. Instead, using a natural experiment from the US they empirically test the effect of a change in the formula design on employment. Their main finding is that a decrease in the formula weight on payroll in one state enhances employment in that state and reduces employment in other states. Hence, they analyze externalities caused by the formula weights and not those caused by the tax rates which are the main interest of our analysis.

There are also models on capital tax competition with unemployment, e.g., Gabszewicz and Ypersele (1996), Lejour and Verbon (1998), Fuest and Huber (1999), Richter and Schneider (2001). But these studies do not consider separate accounting versus formula apportionment. Our model is closest to Ogawa et al. (2004, 2006). They do not analyze formula apportionment, but their framework is similar to separate accounting. One of their main results identifies an unemployment externality. This does not contradict our result that there is no unemployment externality under separate accounting since Ogawa et al. (2004, 2006) consider (implicitly) an endogenous interest rate determined on the world capital market. We deliberately suppose a given interest rate in order to abstract from the unemployment externality already identified in Ogawa et al. (2004, 2006). Our unemployment externality is caused by the specific tax rules under formula apportionment, i.e. consolidation (that induces the tax base effect) and apportionment (that causes the formula effect).
The paper is organized as follows. Section 2 sets up the model which is used in Sections 3 and 4 to study separate accounting and formula apportionment, respectively. Section 5 discusses the robustness of our results if labor is taxed and if unemployment is modeled via an efficiency wage approach. Section 6 concludes.

2 The Model

Multinational Enterprises. There are two small and identical countries denoted by \( a \) and \( b \) and a large number of identical MNEs from which we consider the representative one. In country \( i \in \{a, b\} \), the MNE produces a numeraire good according to the technology \( F(k_i, \ell_i) \). The inputs are mobile capital \( k_i \) and immobile labor \( \ell_i \). The production function \( F \) is homogeneous and has positive and decreasing marginal returns to each input, i.e. \( F_x > 0, F_{xx} < 0 \) for \( x \in \{k_i, \ell_i\} \). It exhibits decreasing returns to scale. Hence, we implicitly assume a fixed third production factor that gives rise to positive rents. Moreover, labor and capital are complements in the sense that \( F_{k\ell} > 0 \), and the production technology is concave, so \( H := F_{kk}F_{\ell\ell} - F_{k\ell}^2 > 0 \).

The MNE may shift profit from one country to the other by, for example, misreporting transfer prices or manipulating its debt-equity structure (e.g. Hines 1999, Clausing 2003). These activities are summarized in the variable \( s \). If \( s > 0 \), the MNE will shift profit from the subsidiary in country \( a \) to the subsidiary in country \( b \). For \( s < 0 \) shifting is the other way round. Profit shifting comes at a concealment cost represented by the function \( C(s) \). This cost reflects, for example, the risk of being detected when illegally shifting profit or the cost of hiring tax consultants (Kant, 1988, Haufler and Schjelderup, 2000). It is supposed to satisfy \( C(0) = C''(0) = 0 \), sign \( C'(s) = \text{sign} \ s \) and \( C''(s) > 0 \). Hence, it is U-shaped with a minimum at \( s = 0 \) when no profit is shifted.

Economic profit of the MNE in country \( i \) equals sales (output) less labor and capital costs and adjusted by profit shifting. Due to partial depreciation allowances and/or partial deductibility of equity financing costs, we assume that capital costs may be partial deductible only. If the fraction of deductible capital costs is denoted by \( \rho \in [0, 1] \) the tax base of the MNE in country \( a \) and \( b \), respectively, is given by

\[
\phi_a = F(k_a, \ell_a) - w_a\ell_a - \rho r k_a - s, \quad \phi_b = F(k_b, \ell_b) - w_b\ell_b - \rho r k_b + s, \quad (1)
\]
where \( r \) is the world market interest rate and \( w_i \) is the wage rate in country \( i \).\(^2\) The interest rate is exogenously given since we assume that the two countries are small relative to the rest of the world. The wage rates are fixed due to the labor market imperfections. A detailed description of the labor market is presented below.

**Residents.** Without loss of generality, the mass of residents in each country is normalized to one. Each resident is endowed with one unit of labor which she inelastically supplies at the labor market in her country. Residents are divided into two groups: employed individuals (index \( e \)) and unemployed individuals (index \( u \)). Employed individuals earn wage income \( w_i \) and a share \( \theta_i \in [0, 1] \) of the MNE’s total after-tax profit \( \pi \). Unemployed individuals get profit income only. An individual of type \( j \in \{e, u\} \) in country \( i \) uses its total income to buy a private consumption good in quantity \( x_{ij} \).

The budget constraints of the two types of residents read
\[
x_{ij} = w_i + \theta_i \pi \quad \text{and} \quad x_{iu} = \theta_i \pi,
\]
respectively. Utility of a type \( j \in \{e, u\} \) individual is given by
\[
U(x_{ij}, g_i) = \lambda x_{ij} + V(g_i),
\]
where \( g_i \) is the quantity of a locally provided public good in country \( i \). The function \( V \) satisfies \( V'(g_i) > 0 \) and \( V''(g_i) \leq 0 \). By assuming quasi-linearity we abstract from income effects in the demand for the public good. The parameter \( \lambda \) allows us to consider the case in which governments maximize tax revenue (\( \lambda = 0 \)) instead of welfare (\( \lambda = 1 \)).

**Unemployment.** As labor is immobile, there is a local labor market in each country. On country \( i \)'s labor market, the MNE’s demand \( \ell_i \) meets the residents’ supply. In contrast to previous studies on separate accounting versus formula apportionment referred to in the introduction, we consider labor market imperfections. It is assumed that these imperfections cause wage rate rigidities which prevent the wage rate to fall below a certain level that, in turn, lies above the equilibrium wage rate. Hence, labor supply exceeds labor demand and some residents are involuntarily unemployed. The unemployment rate in country \( i \) equals \( 1 - \ell_i \). Such a fixed wage approach to unemployment is also used by Ogawa et al. (2004, 2006), for instance. It can be motivated

\(^2\)The deductibility parameter \( \rho \) is assumed to be the same for both countries. At least for the formula apportionment case, a motivation for this assumption is that many formula apportionment systems are characterized by a common definition of the corporate tax base, for example, the Canadian and German system and the proposed system for the EU.
by the widespread use of minimum wage rates. In the EU, for example, 22 member states operate nationwide minimum wage rates and the remainder (Denmark, Finland, Italy, Sweden and Germany) have minimum wage rates in specific sectors.  

**Governments.** The government of country $i$ uses the revenue from corporate income taxation in order to provide the local public good. We formally introduce the public budget constraint in the next sections. The objective of country $i$’s government is to maximize its inhabitants’ Utilitarian welfare

$$W^i = (1 - \ell_i)U(x^u_i, g_i) + \ell_iU(x^e_i, g_i)$$

that equals the sum of all residents’ utility. Inserting (2) into (3) yields

$$W^i = \lambda (\theta_i \pi + w_i \ell_i + r \bar{k}_i) + V(g_i).$$

As already mentioned above, by setting $\lambda = 0$ welfare in country $i$ reduces to the utility the residents receive from the local public good and, thus, from tax revenue.

### 3 Separate Accounting

**Profit Maximization of the MNE.** Under separate accounting, corporate income is taxed in the country where the MNE it declares. Denoting by $t_i$ country $i$’s statutory tax rate, the MNE’s total after-tax profit is given by

$$\pi := (1 - t_a)\phi_a + (1 - t_b)\phi_b - r(1 - \rho)(k_a + k_b) - C(s).$$

The MNE maximizes (5) with respect to profit shifting $s$, investment $k_i$ and labor demand $\ell_i$ for $i \in \{a, b\}$. It takes as given the governments’ tax rates $t_a$ and $t_b$. Indicating profit maximizing values by a tilde, the first-order conditions read

$$(1 - t_i)[F_k(\tilde{k}_i, \tilde{\ell}_i) - \rho \nu] - r(1 - \rho) = 0, \quad F_l(\tilde{k}_i, \tilde{\ell}_i) - w_i = 0, \quad t_a - t_b - C'(\tilde{s}) = 0,$$

for $i \in \{a, b\}$. The first condition in (6) shows that for $\rho < 1$ the MNE’s investment decision is distorted by the corporate tax since the marginal return to capital does not match the interest rate, i.e. $F_k \neq r$. In contrast, labor is fully deductible so that

---

the labor demand decision of the MNE remains undistorted according to the second condition in (6). The third condition determines the MNE’s optimal profit shifting volume. If \( t_a > t_b \), the marginal concealment cost will be positive and the firm transfers profit from country \( a \) into country \( b \). If \( t_a < t_b \), shifting will be the other way round.

For the tax competition analysis, the comparative static effects of the countries’ tax policy on the MNE’s behavior are needed. We follow previous studies and pay attention to a symmetric situation in which both countries have the same tax rate \( t_a = t_b =:\ t \). Differentiating (6) and applying the symmetry assumption yields

\[
\frac{\partial \tilde{k}_i}{\partial t_i} = \frac{(F_k - \rho r) F_{k \ell}}{(1 - t) H} \leq 0, \quad \frac{\partial \tilde{l}_i}{\partial t_i} = -\frac{(F_k - \rho r) F_{k \ell}}{(1 - t) H} \leq 0, \tag{7a}
\]

\[
\frac{\partial \tilde{k}_i}{\partial t_j} = \frac{\partial \tilde{l}_i}{\partial t_j} = 0, \quad \frac{\partial \tilde{s}_i}{\partial t_a} = -\frac{\partial \tilde{s}_i}{\partial t_b} = \frac{1}{C''} > 0, \tag{7b}
\]

for \( i, j \in \{a, b\} \) and \( i \neq j \), with \( F_k - \rho r = (1 - \rho)r/(1 - t) \geq 0 \) due to (6). For partial or no deductibility of capital cost \((\rho \in [0, 1])\), equation (7a) shows that increases in country \( i \)'s tax rate reduce both capital and labor demand in country \( i \). The effect on labor demand stems from the complementarity of capital and labor. If capital is fully deductible \((\rho = 1)\), equation (6) simplifies to \( F_k - r = 0 \). In this case, both investment and employment in country \( i \) are independent of country \( i \)'s tax rates. Due to the separate accounting principle, it is clear that tax increases in country \( i \) do not influence investment and employment in country \( j \). This is proven by the first condition in (7b). The second condition in (7b) shows that an increase in one country’s tax rate will induce the MNE to shift more profit into the other country.

**Tax Competition and Fiscal Externalities.** Country \( i \)'s budget constraint reads \( g_i = t_i \phi_i \). It equates tax revenue and the expenditure for the public good. Country \( i \)'s government chooses the tax rate \( t_i \) in order to maximize (4) subject to \( g_i = t_i \phi_i \). In doing so, it takes as given country \( j \)'s tax rate \( t_j \) and anticipates the MNE’s behavior represented by (6) or, equivalently, (7a) and (7b). The first-order condition \( \partial W^i / \partial t_i = 0 \) determines country \( i \)'s reaction function, i.e. its best response to country \( j \)'s tax rate.\(^4\)

\(^4\)The second-order condition requires that the welfare function \( W^i \) is strictly concave in \( t_i \). While it is not possible to prove this as a general result, in an earlier version of this paper we run a number of numerical simulations where the second-order condition is satisfied.
As already mentioned above, we focus on a symmetric Nash equilibrium with equilibrium tax rates $\tilde{t}_a = \tilde{t}_b =: \tilde{t}$. Symmetry requires that the fixed wage rate is equal in both countries and the residents of both countries own the same share of the MNE, i.e. $w_a = w_b =: w$ and $\theta_a = \theta_b =: \theta$. Due to (1) and (6) the MNE then realizes the same investment, labor demand and tax base in both countries, i.e. $\tilde{k}_a = \tilde{k}_b =: \tilde{k}$, $\tilde{\ell}_a = \tilde{\ell}_b =: \tilde{\ell}$ and $\phi_a = \phi_b =: \phi$. Our main interest is to assess the efficiency properties of the equilibrium tax rates. This can be done by investigating the fiscal externalities, i.e. the effect of country $i$’s tax rate on welfare in country $j$. A positive (negative) fiscal externality implies inefficient undertaxation (overtaxation). We report on the effect of $t_b$ on $W^a$ only, since the symmetry assumption ensures that the effect of $t_a$ on $W^b$ is the same. Differentiating (4) and taking into account (1), (7b) and $g_a = t_a \phi_a$, we obtain

$$\frac{\partial W^a}{\partial t_b} = \lambda \theta \frac{\partial \pi}{\partial t_b} - iV'(\cdot) \frac{\partial \tilde{s}}{\partial t_b}. \quad (8)$$

There are two cross effects of country $b$’s tax rate on country $a$’s welfare. First, increasing $t_b$ reduces the MNE’s after-tax profit and, thus, profit income of country $a$’s residents. This income externality is represented by the first term on the RHS of (8). It is negative since the envelope theorem implies $\lambda \theta \frac{\partial \pi}{\partial t_b} = -\lambda \theta \phi < 0$. Second, if country $b$ raises its tax rate, profit will be shifted from $b$ to $a$ with the consequence that the tax revenue and the quantity of the public good in country $a$ go up. This profit shifting externality is reflected by the second term on the RHS of (8). It is positive due to (7b). Since the income externality disappears if $\lambda = 0$, we immediately obtain

**Proposition 1.** Suppose the tax competition game under separate accounting attains a symmetric Nash equilibrium with $\tilde{t}_a = \tilde{t}_b =: \tilde{t}$. If governments maximize tax revenue ($\lambda = 0$), the equilibrium tax rate $\tilde{t}$ will be inefficiently low. Under welfare maximization ($\lambda > 0$), in contrast, the equilibrium tax rate $\tilde{t}$ may be inefficiently low or high.

These insights coincide with those derived by previous studies under the assumption of perfect labor markets. Hence, the contribution of Proposition 1 is to show that the results of previous studies remain true when labor market imperfections are taken into account. Put differently, our analysis yields the new insight that unemployment itself does not create additional fiscal externalities under separate accounting.
4 Formula Apportionment

Profit Maximization of the MNE. Under formula apportionment, tax bases are consolidated and then distributed to the two countries according to a certain formula. In practice, the formula usually employs three apportionment factors in convex combinations: the capital, sales and payroll shares of the MNE in the respective country. Denoting by $\gamma$, $\sigma$ and $\varphi$ the weights these factors receive in the formula, the share of the consolidated tax base that is assigned to country $a$ equals

$$A(k_a, k_b, \ell_a, \ell_b) = \gamma \frac{k_a}{k_a + k_b} + \sigma \frac{F(k_a, \ell_a)}{F(k_a, \ell_a) + F(k_b, \ell_b)} + \varphi \frac{w_a \ell_a}{w_a \ell_a + w_b \ell_b},$$

(9)

with $(\gamma, \sigma, \varphi) \in S$ where $S := \{(\gamma, \sigma, \varphi) \mid (\gamma, \sigma, \varphi) \in [0, 1]^3$ and $\gamma + \sigma + \varphi = 1\}$ denotes the set of all feasible weights. The share $1 - A(\cdot)$ of the consolidated tax base remains for country $b$.

The MNE’s tax burden in the countries $a$ and $b$ is given by $t_a A(\cdot) (\phi_a + \phi_b)$ and $t_b [1 - A(\cdot)] (\phi_a + \phi_b)$, respectively. The MNE’s total after-tax profit reads

$$\pi = (1 - \tau)(\phi_a + \phi_b) - r(1 - \rho)(k_a + k_b) - C(s),$$

(10)

where $\tau = t_a A(k_a, k_b, \ell_a, \ell_b) + t_b [1 - A(k_a, k_b, \ell_a, \ell_b)]$ is the effective (or weighted average) tax rate that the MNE faces in country $a$ and $b$.

The MNE maximizes (10) with respect to capital, labor and profit shifting taking into account (1), (9) and the definition of $\tau$. Indicating profit maximizing values by a hat, with respect to profit shifting we obtain $C'(\hat{s}) = 0$ and, thus, $\hat{s} = 0$. Due to consolidation, the MNE does not shift profit. The other first-order conditions are

$$-(t_a - t_b)A_{k_i}(\cdot)(\phi_a + \phi_b) + (1 - \tau)[F_k(\hat{k}_i, \hat{\ell}_i) - \rho r] - r(1 - \rho) = 0, \quad (11a)$$

$$-(t_a - t_b)A_{\ell_i}(\cdot)(\phi_a + \phi_b) + (1 - \tau)[F_{\ell_i}(\hat{k}_i, \hat{\ell}_i) - w_i] = 0, \quad (11b)$$

for $i \in \{a, b\}$. Similar to separate accounting, (11a) and (11b) contain the marginal return to capital and labor and the respective factor cost. But there is now an additional term that reflects the MNE’s formula manipulation incentive. For a nonzero tax rate differential $t_a - t_b$ the MNE invests more and demands more labor in the country with the lower tax rate since this gives the lower tax rate a higher weight in the calculation of the effective tax rate and, thus, reduces the total tax liability.

5We assume that both countries use the same formula, so the two shares add up to one. This is not the case in the U.S., but in Canada, Germany and in the discussed EU apportionment system.
In order to figure out the comparative static effects of tax rate changes on the MNE’s behavior, we again focus on a symmetric solution with equal tax rates. We then have \( t_a = t_b = \tau = t, \hat{k}_a = \hat{k}_b =: \hat{k}, \hat{\ell}_a = \hat{\ell}_b =: \hat{\ell}, \phi_a = \phi_b =: \phi, A(\cdot) = 1/2 \) and

\[
A_{k_a} = -A_{k_b} = \frac{\gamma}{4k} + \frac{\sigma F_k}{4F}, \quad A_{\ell_a} = -A_{\ell_b} = \frac{\phi}{4\ell} + \frac{\sigma F_\ell}{4F}.
\]  

(12)

Total differentiating (11a) and (11b), the Appendix proves

\[
\frac{\partial \hat{k}_i}{\partial t_i} = \frac{1}{2(1-t)H} \left[ (F_k - \rho r)F_{k\ell} + \phi \left( \frac{\gamma F_{k\ell}}{k} + \frac{\sigma(F_kF_{k\ell} - F_\ell F_{k\ell})}{F} - \frac{\varphi F_{k\ell}}{\ell} \right) \right], \quad (13a)
\]

\[
\frac{\partial \hat{\ell}_i}{\partial t_i} = \frac{1}{2(1-t)H} \left[ -(F_k - \rho r)F_{k\ell} - \phi \left( \frac{\gamma F_{k\ell}}{k} + \frac{\sigma(F_kF_{k\ell} - F_\ell F_{k\ell})}{F} + \frac{\varphi F_{k\ell}}{\ell} \right) \right], \quad (13c)
\]

\[
\frac{\partial \hat{k}_j}{\partial t_i} = \frac{1}{2(1-t)H} \left[ -(F_k - \rho r)F_{k\ell} - \phi \left( \frac{\gamma F_{k\ell}}{k} + \frac{\sigma(F_kF_{k\ell} - F_\ell F_{k\ell})}{F} - \frac{\varphi F_{k\ell}}{\ell} \right) \right], \quad (13b)
\]

\[
\frac{\partial \hat{\ell}_j}{\partial t_i} = \frac{1}{2(1-t)H} \left[ (F_k - \rho r)F_{k\ell} + \phi \left( \frac{\gamma F_{k\ell}}{k} + \frac{\sigma(F_kF_{k\ell} - F_\ell F_{k\ell})}{F} - \frac{\varphi F_{k\ell}}{\ell} \right) \right], \quad (13d)
\]

with \( i \in \{a,b\}, i \neq j \), and \( F_k - \rho r = (1 - \rho)/(1 - t) \geq 0 \) according to (11a) and the symmetry property. The comparative static effects of country \( i \)'s tax rate in (13a)-(13d) can be decomposed into two partial effects. The first is a tax base effect reflected by the terms containing \( F_k - \rho r \). Under no or partial deductibility of capital cost \( (\rho \in [0,1]) \), increasing country \( i \)'s tax rate raises the MNE’s effective tax burden and thus induces the MNE to reduce the tax base by lowering capital and labor demand in both countries. Even though labor cost is fully deductible, the tax base effect is also present at labor since labor is complementary to capital. The tax base effect emerges in both countries since tax bases are consolidated and taxed at the effective tax rate. Under full deductibility of capital cost, the tax base effect disappears since for \( \rho = 1 \) it holds \( F_k - \rho r = (1 - \rho)/(1 - t) = 0 \). The second partial effect of country \( i \)'s tax rate is based on the above-mentioned incentive of the MNE to manipulate the apportionment formula. If country \( i \) increases its tax rate, the MNE will reallocate capital and labor from country \( i \) to country \( j \). This formula effect is captured by the terms containing the formula weights \( \gamma, \sigma \) and \( \varphi \). It is a pure reallocation effect since in (13a) and (13b) and in (13c) and (13d) it is equal in size but opposite in sign. Hence, the formula effect does not play a role for the impact of country \( i \)'s tax rate on the MNE’s total investment and labor demand, as is easily seen by adding (13a) to (13b) and (13c) to...
In the aggregate, only the tax base effect remains, so the MNE reduces total investment and labor demand as reaction upon an increase in country \( i \)'s tax rate.

There is an important difference of these comparative static effects to those under separate accounting. Under both tax principles, changes in country \( i \)'s tax rate affect unemployment in country \( i \), see (7a) and (13c). According to (7b) and (13d), in contrast, the effect of country \( i \)'s tax rate on unemployment in country \( j \) is non-zero only under formula apportionment. This difference will be important when we now turn to the fiscal externalities arising from tax competition under formula apportionment.

**Tax Competition and Fiscal Externalities.** Under formula apportionment, the fiscal budget of country \( a \) and \( b \), respectively, satisfies the constraint

\[
g_a = t_a A(\hat{k}_a, \hat{k}_b, \hat{\ell}_a, \hat{\ell}_b)(\phi_a + \phi_b), \quad g_b = t_b[1 - A(\hat{k}_a, \hat{k}_b, \hat{\ell}_a, \hat{\ell}_b)](\phi_a + \phi_b),
\]

where capital and labor depend on the tax rates via (13a)–(13d). The countries are again engaged in tax competition and set the tax rates such that welfare (4) is maximized subject to the budget constraint in (14). In doing so, they take as given the tax rate of the respective other country. The Nash equilibrium of the tax competition game is again determined by \( \partial W^i / \partial t_i = 0 \) for \( i \in \{a, b\} \).

To study the efficiency properties of the tax rates in the Nash equilibrium we restrict our attention again to symmetric equilibria with tax rates \( \hat{t}_a = \hat{t}_b = \tau =: \hat{t} \). The fiscal externalities of country \( b \)'s tax rate on country \( a \)'s welfare are then reflected by

\[
\frac{\partial W^a}{\partial \hat{t}_b} = \text{IE} + \text{TE} + \text{FE}|_{(\gamma, \sigma, \phi)} + \text{UE}|_{(\gamma, \sigma, \phi)},
\]

with

\[
\text{IE} = -\lambda \theta \phi < 0, \quad \text{TE} = \frac{\hat{t}V'(\cdot) (F_k - \rho r) \partial(\hat{k}_a + \hat{k}_b)}{2} \frac{\partial}{\partial \hat{t}_b} \leq 0, \quad (16a)
\]

\[
\text{FE}|_{(\gamma, \sigma, \phi)} = 2\hat{t} \phi V'(\cdot) \left[ A_{\hat{k}_a} \frac{\partial(\hat{k}_a - \hat{k}_b)}{\partial \hat{t}_b} + A_{\hat{\ell}_a} \frac{\partial(\hat{\ell}_a - \hat{\ell}_b)}{\partial \hat{t}_b} \right] > 0, \quad (16b)
\]

\[
\text{UE}|_{(\gamma, \sigma, \phi)} = \lambda w \frac{\partial \hat{\ell}_a}{\partial \hat{t}_b}, \quad (16c)
\]

where the signs of (16a) and (16b) follow from (13a)–(13d). Due to the consolidation of tax bases the profit shifting externality disappears under formula apportionment.
However, the income externality in (16a) remains and tends to inefficient undertaxation. In addition, several other externalities emerge under formula apportionment. The externalities in (16a) and (16b) go back to the tax base and formula effects derived above. If country b increases its tax rate, it does not take into account that country a’s tax revenue declines due to a reduction in the MNE’s consolidated tax base. This constitutes a negative tax base externality (TE) which points to inefficient overtaxation. Moreover, in raising its tax rate country b ignores that country a’s tax revenue increases since the MNE reallocates production factors from country b to country a so that a larger share of the consolidated tax base is taxed in country a. This represents a positive formula externality (FE) and tends to inefficient undertaxation. Income, tax base and formula externalities are also derived by, for example, Nielsen et al. (2004) and Riedel and Runkel (2007). But they use a model without labor and labor market imperfections. Hence, our analysis shows that income, tax base and formula externalities also emerge in the presence of unemployment.

The main contribution of our analysis, however, is a fourth externality, in (16c). This externality is caused by unemployment and, therefore, is absent in previous studies that abstract from labor market imperfections. The underlying intuition is straightforward: If country b increases its tax rate, the MNE will change its labor demand not only in country b but also in country a as shown by (13d). Country b’s tax rate thus influences unemployment and welfare in country a. The unemployment externality arises since country b ignores this cross country effect of its tax policy. Denoting the substitution elasticity of the production function by , the unemployment externality is characterized by the following proposition proven in the Appendix.

**Proposition 2.** For any formula in we have . Moreover, for a given tax rate it holds:

(i) \[
\max_{(\gamma,\sigma,\varphi) \in \mathcal{S}} \text{UE}_{(\gamma,\sigma,\varphi)} = \text{UE}_{(0,0,1)}.
\]

(ii) If , then \[
\min_{(\gamma,\sigma,\varphi) \in \mathcal{S}} \text{UE}_{(\gamma,\sigma,\varphi)} = \text{UE}_{(0,1,0)}.
\]

(iii) If , then \[
\min_{(\gamma,\sigma,\varphi) \in \mathcal{S}} \text{UE}_{(\gamma,\sigma,\varphi)} = \text{UE}_{(1,0,0)}.
\]

Since the above-mentioned tax base effect is independent of the formula weights and , the intuition of these results goes back to the formula effect. Moreover, we
have to take into account that the formula transforms the corporate income tax into a
tax on the apportionment factors (e.g. McLure, 1980). If the apportionment formula
contains the payroll factor only, the tax burden on labor is therefore higher than for any
other formula. It follows that the formula effect – and, thus, the cross-country effect
of tax rates on labor as well as the unemployment externality – is the highest under
a pure payroll formula, see Proposition 2 (i). The tax burden on labor is the lowest
if the formula contains the property factor only. However, this does not imply that
the unemployment externality is minimized for the pure property formula. We have to
take into account that a change in capital is always associated with a change in labor
demand and that the extent of this change depends on the substitution elasticity of the
production function.6 If capital and labor are complements in the sense that \( \eta \in [0,1] \),
an increase in capital is associated with a large increase in labor. The cross-country
effect of tax rates on labor and the unemployment externality are then large, even if
the apportionment formula uses property only. The minimum of the unemployment
externality is therefore attained with the pure sales formula instead of the pure capital
formula, as shown in Proposition 2 (ii). This argument is reversed if capital and labor
are substitutes in the sense that \( \eta \in [1,\infty[ \). The unemployment externality is then
minimized for the pure property formula, as can be seen from Proposition 2 (iii). Pure
sales and property formulas both minimize the unemployment externality in the special
case of the Cobb-Douglas production function (\( \eta = 1 \)). In any case, the unemployment
externality is positive and, thus, tends to inefficient undertaxation.

Interplay of Income and Unemployment Externalities. As the sign of the
unemployment externality may be different from that of the other externalities, it is
important to investigate the interplay between the different externalities. We start
with the comparison between the externalities that work through the private income
of households, i.e. unemployment and income externalities. Denoting by \( \beta := \ell F_\ell/F \)
the production elasticity of labor, the Appendix proves

**Proposition 3.** (i) If \( \eta \in [0,1] \), then for given tax rate \( \hat{t} \) we have \( \text{IE} + \text{UE} \) \( \gamma,\sigma,\phi \) > 0

---

6Formally, symmetry implies that (11b) can be written as \( F_\ell(\hat{k}_i, \hat{\ell}_i) - w_i = 0 \). If capital \( \hat{k}_i \) increases
due to an increase in \( t_j \), then \( F_{k\ell} > 0 \) implies that \( \hat{\ell}_i \) increases, too. The extent of this effect is
determined by the size of \( F_{k\ell} \) which, in turn, is determined by the substitution elasticity.
for all \((\gamma, \sigma, \varphi) \in S\) if and only if
\[
\beta > \frac{\mu \Psi - \sqrt{\mu^2 \Psi^2 + 4 \mu (1 - \eta) (1 - \rho) (1 - \hat{t}) [(1 - \mu) (1 - \rho \hat{t}) + \mu (1 - \rho)]}}{2 (\eta - 1) (1 - \rho)} =: \bar{\beta}_1, \quad (17)
\]
with \(\Psi := \eta (2 - \hat{t} - \rho) + (1 - \eta) (1 - \hat{t}) > 0\).

(ii) If \(\eta \in [1, \infty[\), then for given tax rate \(\hat{t}\) we have \(\text{IE} + \text{UE}|_{(\gamma, \sigma, \varphi)} > 0\) for all \((\gamma, \sigma, \varphi) \in S\) if and only if
\[
\beta > \frac{\mu (1 - \hat{t}) [(1 - \mu) (1 - \rho \hat{t}) + \mu (1 - \rho)]}{(1 - \eta + \mu \eta) (1 - \rho \hat{t}) + \mu (1 - \rho) (1 - \hat{t})} =: \bar{\beta}_2. \quad (18)
\]

Proposition 3 shows that the unemployment externality will be large enough to compensate the income externality if the production elasticity of labor, \(\beta\), is sufficiently large, i.e. larger than the threshold values \(\bar{\beta}_1\) and \(\bar{\beta}_2\) in (17) and (18), respectively. This result is plausible as the size of the unemployment externality is not only increasing in the formula weight placed on payroll but also in the production elasticity \(\beta\). The higher this elasticity, the more productive is labor and the larger is the cross-country effect of tax rates on labor and unemployment. It is then clear that for a high production elasticity of labor the positive unemployment externality overcompensates the negative income externality and the sum of both externalities points to inefficient undertaxation.

Whether (17) and (18) are satisfied depends \(\textit{inter alia}\) on the properties of the production function. The empirical literature provides several, partially conflicting, results on these properties. In a growth context, Duffy and Papageorgiou (2000) find \(\mu = 0.99\) and \(\eta = 2.27\), so (18) is the relevant condition. From the results of Devereux et al. (2002) and Desai et al. (2004) we can compute \(\rho = 0.69\).\(^7\) It remains to specify \(\hat{t}\). This is an endogenous variable in our model, so we cannot use estimations from the previous literature. But with \(\mu = 0.99\), \(\eta = 2.27\) and \(\rho = 0.69\) it can be shown that the threshold value \(\bar{\beta}_2\) in (18) is decreasing in \(\hat{t}\) and smaller than 0.25 for all \(\hat{t} \in [0, 1]\). As \(\beta\) is usually estimated to be between 0.35 and 0.45 (Mankiw et al., 1992; Nonneman and Vanhoudt, 1996), condition (18) will be satisfied for all possible values of \(\hat{t}\) and the sum of unemployment and income externalities is positive. This argument is robust against variations in the value of the substitution elasticity. In a public economics context authors often use a much lower value (for a survey see Chirinko, 2002). A

\(^7\)See Riedel and Runkel (2007) for details on this computation.
popular value is $\eta = 0.4$. Then (17) is the relevant condition and the threshold value $\bar{\beta}_1$ is smaller than 0.27 for all $\hat{t} \in [0, 1]$. For the Cobb-Douglas case with $\eta = 1$, the threshold $\bar{\beta}_1$ is smaller than 0.24. It is worth mentioning that also variations in the degree of returns to scale, $\mu$, do not alter the threshold values very much. Hence, from an empirical point of view it seems likely that the conditions in Proposition 3 are satisfied and that the sum of unemployment and income externality is positive.

**Interplay of Tax Base and Formula Externalities.** We now turn to the externalities that work through the public budgets. In contrast to the above analysis that employs a general production function, it is now useful to start with the special case of the Cobb-Douglas technology, i.e. $F(k_i, \ell_i) = k_i^\alpha \ell_i^\beta$ with $\alpha, \beta > 0$ and $\alpha + \beta \in ]0, 1[$. This function is obtained from the general production function if we set $\eta = 1$ and $\mu = \alpha + \beta$. Note that $\beta$ still equals the production elasticity of labor $\ell F/\ell$. At the end of this section we will briefly discuss generalizations of the production function.

Using the Cobb-Douglas specification, the Appendix shows

**Proposition 4.** Suppose the production technology is Cobb-Douglas, i.e. $F(k_i, \ell_i) = k_i^\alpha \ell_i^\beta$ with $\alpha + \beta \in ]0, 1[$. Then for given tax rate $\hat{t}$ it holds:

(i) $\min_{(\gamma, \sigma, \varphi) \in S} \text{FE}|_{(\gamma, \sigma, \varphi)} = \text{FE}|_{(0,1,0)} > 0$.

(ii) If capital cost is either fully deductible ($\rho = 1$) or not deductible at all ($\rho = 0$), then $\text{TE} + \text{FE}|_{(\gamma, \sigma, \varphi)} > 0$ for all $(\gamma, \sigma, \varphi) \in S$.

With a pure property or payroll formula, the MNE’s incentive to manipulate the formula is quite strong since apportionment is targeted directly at the production factors. As a consequence, the formula externality is relatively large. In contrast, if apportionment uses the sales share only, then it will be directed at the production factors indirectly, namely via the production function. Due to the existence of a fixed factor, manipulating a pure sales formula is more difficult than manipulating other formulas. This is the reason why the formula externality is minimized under a pure sales formula as shown by Proposition 4 (i). Nevertheless, Proposition 4 (ii) states that, at least for a very high or a very low deductibility of capital cost, the formula externality is still large enough to overcompensate the negative tax base externality so that the sum of both externalities is positive and tends to inefficient undertaxation. Haufler (2006) obtained
the same result. But he used a model with capital as the sole variable production and apportionment factor. Hence, our analysis shows that the result can be generalized to the case with labor as the second variable production factor and with unemployment.

**Interplay of all Externalities.** To judge the overall efficiency of the equilibrium tax rates, we have to clarify the sign of the sum of externalities. Proposition 3 and 4 together with the Cobb-Douglas specification \( \eta = 1 \) immediately imply

**Proposition 5.** Suppose the tax competition game under formula apportionment attains a symmetric Nash equilibrium with \( \hat{t}_a = \hat{t}_b =: \hat{t} \). Moreover, suppose the production technology is Cobb-Douglas, i.e. \( F(k_i, \ell_i) = k_i^\alpha \ell_i^\beta \) with \( \alpha + \beta \in [0, 1] \), and capital cost is either fully deductible \( (\rho = 1) \) or not deductible at all \( (\rho = 0) \). If

(i) governments maximize tax revenues \( (\lambda = 0) \) or

(ii) the production elasticity of labor \( \beta \) satisfies (18),

then \( \hat{t} \) will be inefficiently low for all \( (\gamma, \sigma, \varphi) \in S \).

Proposition 5 identifies sufficient conditions for the equilibrium tax rates under formula apportionment to be inefficiently low. According to Proposition 5 (i), undertaxation occurs if governments maximize tax revenue. In this case, income and unemployment externalities are absent. The sum of the remaining tax base and formula externalities is positive under the conditions of Proposition 4 (ii) and, thus, causes inefficiently low tax rates. For the case of welfare maximization, Proposition 5 (ii) proves undertaxation if the production elasticity of labor is sufficiently large so that the sum of income and unemployment externalities is positive due to Proposition 3. It is worth pointing out the important role the unemployment externality plays for this result. While the sum of tax base and formula externalities is positive, the income externality has a negative sign. Without unemployment the income externality may therefore cause inefficient overtaxation. But taking into account labor market imperfections, the unemployment externality overcompensates the income externality and ensures that tax rates fall short of their efficient level. In other words, under the conditions of Proposition 5 overtaxation is possible without unemployment but not with unemployment.

Propositions 2 and 4 show that the size of the unemployment and the formula externalities depends on the shape of the apportionment formulas. Hence, starting
from the insight that the equilibrium tax rate under formula apportionment may be inefficient, it is natural to ask under which formula the tax rate comes closest to the efficient solution. To answer the question, it is useful to rank the equilibrium tax rate under the different formulas. In doing so, we need the first-order condition of the countries’ welfare maximization. Maximizing (4) subject to (14) and using (16a)–(16c) as well as 
\[ \frac{\partial \hat{\ell}_b}{\partial \tau_b} = -\frac{\partial \hat{\ell}_a}{\partial t_b} - (F_k - \rho r) F_{kt}/[(1 - \hat{t})H] \]
from (13c) and (13d), the first-order condition for country $b$’s welfare maximum can be written as

\[ \frac{\partial W^b}{\partial t_b} \bigg|_{(\gamma,\sigma,\varphi)} = \phi V' - \lambda w \frac{(F_k - \rho r) F_{kt}}{(1 - \hat{t})H} + IE + TE - FE|_{(\gamma,\sigma,\varphi)} - UE|_{(\gamma,\sigma,\varphi)} = 0. \quad (19) \]

The first four terms in (19) are independent of the formula weights since in a symmetric equilibrium with equal tax rates neither investment nor labor demand depends on the formula weights according to (11a) and (11b). Comparing the last two terms for different formulas reveals which of the formulas implements the highest equilibrium tax rate. Proposition 2 (ii) and Proposition 4 (i) state that in the Cobb-Douglas case both $FE|_{(\gamma,\sigma,\varphi)}$ and $UE|_{(\gamma,\sigma,\varphi)}$ are positive and minimized at the pure sales formula. Consequently, if the welfare function is assumed to be concave, we obtain

**Proposition 6.** Suppose the tax competition game under formula apportionment attains a symmetric Nash equilibrium and the production technology is Cobb-Douglas, i.e. $F(k_i, \ell_i) = k_i^\alpha \ell_i^\beta$ with $\alpha + \beta \in [0, 1]$. Then the equilibrium tax rate under the sales formula $(\gamma, \sigma, \varphi) = (0, 1, 0)$ is higher than under any other formula $(\gamma, \sigma, \varphi) \in S \backslash \{(0, 1, 0)\}$.

If the whole formula weight lies on capital, the countries possess a strong incentive for lowering the tax rates since in this case they try to attract more capital and increase welfare. The same is true if the whole weight is placed on payroll since then a decrease in the tax rate ceteris paribus reduces unemployment and increases welfare. From these two polar cases it is intuitively clear that the incentive for a race-to-the-bottom in corporate income taxation is the weakest under a pure sales formula as shown by Proposition 6. If the formula exclusively uses the sales factor, the incentive for lowering tax rates will be weakened since, in contrast to pure property or payroll formulas, the formula now contains a fixed element via the fixed production factor.

With the help of this insight, we can now answer the question which formula generates the smallest distortion of corporate taxation. Propositions 5 and 6 imply
Proposition 7. Suppose all assumptions of Proposition 5 are satisfied. Then the equilibrium tax rate comes closer to the efficient tax rate under the pure sales formula \((\gamma, \sigma, \varphi) = (0, 1, 0)\) than under any other formula \((\gamma, \sigma, \varphi) \in S \setminus \{(0, 1, 0)\}\).

Under the conditions of Proposition 5, we have inefficient undertaxation and from Proposition 6 we know that under the sales formula tax rates are the highest. It is therefore clear that the sales formula performs best among all feasible apportionment mechanisms. The sales formula ensures the best mix of the counteracting externalities. It ensures that the (positive) sum of the formula and tax base externalities is minimized and it generates an unemployment externality of ideal size. This means that the unemployment externality is strong enough to overcompensate the income externality, but not too strong to push the equilibrium tax rate far away from its efficient level.

The Role of the Production Technology. The results in Propositions 2 and 3 are true for a general production function, whereas Propositions 4–7 are derived under the assumption of a Cobb-Douglas technology. We will now briefly discuss the robustness of the latter propositions against generalizations of \(F\). For this we consider the CES specification \(F(k_i, \ell_i) = \frac{Q^\mu}{\nu} = \left[\left(1 - (\delta k)^\mu \ell^\nu\right) \left(1 - (\delta k)^\mu \ell^\nu\right)\right]^{\frac{\mu}{\nu}}\) with \(\mu \in [0, 1]\) and \(\nu < 1\). The substitution elasticity is then given by \(\eta = \frac{1}{(1 - \nu)}\). Hence, increasing the parameter \(\nu\) is equivalent to increasing the substitution elasticity \(\eta\).

With the help of the derivatives \(F_k = \mu KQ^\mu/\hat{k}, F_\ell = \mu LQ^\mu/\hat{\ell}, F_{kk} = -\mu[(1 - \mu)K + (1 - \nu)\hat{k}^2], F_{\ell\ell} = -\mu[(1 - \mu)L + (1 - \nu)\hat{\ell}^2]\) and \(F_{k\ell} = \mu(\mu - \nu)KLQ^\mu/\hat{k}\hat{\ell}\) it can be shown, analogous to Proposition 4 (i), that among all pure formulas the formula externality \(FE|_{(\gamma, \sigma, \varphi)}\) is minimized for the pure sales formula \((\gamma, \sigma, \varphi) = (0, 1, 0)\). The formula and tax base externalities can then be written as

\[
FE|_{(0,1,0)} = \Gamma \left(\frac{K^2}{1 - \mu} + 2KL + (1 - \mu)\ell^2\right) > 0, \quad TE = -\Gamma \left(\frac{K^2}{1 - \mu} + \frac{KL}{1 - \nu}\right) < 0.
\]

with \(\Gamma := \mu \hat{\nu} Q^\mu/\left(2(1 - \hat{\ell})\right) > 0\). Summing up implies

\[
TE + FE|_{(0,1,0)} \gtrless 0 \iff \nu \gtrless \frac{1}{2} + \frac{(1 - \mu)L}{4K + 2(1 - \mu)L} := \bar{\nu},
\]

where \(\nu \in [0, 1]\). In contrast to the Cobb-Douglas case in Proposition 4 (ii), we now cannot prove as a general result that the sum of tax base and formula externalities is positive. If the substitution elasticity is larger than a threshold value (i.e. \(\nu > \bar{\nu}\), the
tax base externality is rather large in absolute terms and overcompensates the formula externality. From an empirical point of view, however, it seems unlikely that the condition \( \nu > \bar{\nu} \) is satisfied. As already stated above, in public finance studies a widely used value of the substitution elasticity is \( \eta = 0.4 \). This value implies \( \nu = (\eta - 1)/\eta = -1.5 \), so the sum of tax base and formula externalities is positive according to (21). We can conclude that Proposition 4 and, thus, the undertaxation result in Proposition 5 is likely to remain true under the assumption of a more general production function.

And also the results derived in Propositions 6 and 7 hold for a broader class of production functions. Proposition 2 (ii) states that the unemployment externality is minimized for the pure sales formula whenever the substitution elasticity \( \eta \) lies in the interval \([0, 1]\). Hence, Proposition 6 is true not only for the Cobb-Douglas production function, but for all production functions with \( \eta \in [0, 1] \) or, equivalently, \( \nu \leq 0 \). In combination with the above derived result that Proposition 5 is true for all CES production functions with \( \nu \leq \bar{\nu} \) it follows that Proposition 7 holds for all CES production functions with \( \nu \leq 0 \). Again, this latter condition is consistent with the value \( \nu = -1.5 \) (\( \eta = 0.4 \)) that is often used in empirical public finance studies.

5 Extensions

In this section we consider two important extensions to our analysis in the previous sections. We first introduce a labor tax into the analysis and then discuss an efficiency wage framework as an alternative approach to unemployment.\(^9\)

**Labor Tax.** One may ask whether the existence of the unemployment externality is only due to the fact that so far we focused on corporate income taxation only and ignored labor taxation. However, we will now show that introducing labor taxation creates additional externalities instead of removing the unemployment externality.

Suppose the government in country \( i \) levies a tax \( T_i \) on the MNE’s labor input \( \ell_i \) in country \( i \). The tax bases of the MNE in country \( a \) and \( b \) then changes to

\[
\phi_a = F(k_a, \ell_a) - (w_a + T_a)\ell_a - \rho r k_a - s, \quad \phi_b = F(k_b, \ell_b) - (w_b + T_b)\ell_b - \rho r k_b + s. \tag{22}
\]

\(^8\)For a substitution elasticity of \( \eta = 2.27 \) derived by Duffy and Papageorgiou (2000) we obtain \( \nu \approx 0.515 \). This value lies only slightly above 0.5 so that the condition \( \nu < \bar{\nu} \) may still be satisfied.

\(^9\)We here report on the results only. The proofs can be obtained from the authors upon request.
Replacing (1) by (22), under symmetry \((t_a = t_b \text{ and } T_a = T_b)\) the comparative static effects of labor taxes on the MNE’s behavior are the same under separate accounting and formula apportionment and given by

\[
\frac{\partial \ell_i}{\partial T_i} = \frac{F_{kk} H}{H} < 0, \quad \frac{\partial k_i}{\partial T_i} = -\frac{F_{k\ell} H}{H} < 0, \quad \frac{\partial \ell_i}{\partial T_j} = \frac{\partial k_i}{\partial T_j} = \frac{\partial s}{\partial T_i} = 0, \quad (23)
\]

with \(k \in \{\tilde{k}, \hat{k}\}, \ell \in \{\tilde{\ell}, \hat{\ell}\}, s \in \{\tilde{s}, \hat{s}\}, i, j \in \{a, b\} \text{ and } i \neq j\). An increase in country \(i\)’s labor tax reduces employment and investment in this country. Since the labor demand in country \(i\) is not affected by country \(j\)’s labor tax, investment in country \(i\) and profit shifting are independent of country \(j\)’s labor tax, too.

Since there are no cross-country effects of labor taxes on investment, labor demand and profit shifting, profit income is the only channel through which welfare in one country is affected by the labor tax of the other country. Hence, under both separate accounting and formula apportionment we obtain the negative income externality

\[
\frac{\partial W^a}{\partial T_b} = IE_{\ell} = -\lambda \theta (1 - t) \ell < 0, \quad (24)
\]

with \(t \in \{\tilde{t}, \hat{t}\} \text{ and } \ell \in \{\tilde{\ell}, \hat{\ell}\}\). Under both taxation principles, labor tax rates are therefore inefficiently low. This result shows that the labor tax cannot be used to internalize the unemployment externality. On the contrary, the labor tax itself creates a fiscal externality and contributes to the inefficiency of international taxation.

**Efficiency Wage Approach to Unemployment.** As argued above, there is evidence that a fixed wage rate approach to unemployment is relevant. Nevertheless, it is also known that there may be other reasons for unemployment. We therefore now discuss how our results are changed when we consider the efficiency wage approach to unemployment developed by Shapiro and Stiglitz (1984).\(^{10}\)

The main difference of this approach to the fixed wage approach is that workers decide on an effort level which determines the probability of being fired at their present position. Suppose we have a continuum of identical MNEs. The mass of MNEs is normalized to one. Consider household \(h \in [0, 1]\) in country \(i\) employed in MNE \(m \in [0, 1]\) earning the wage rate \(w_m^i\). The household inelastically supplies one unit of labor and chooses an effort level \(e_{hi}^m \in [0, 1]\) at effort costs \((1 + 1/\varepsilon)(e_{hi}^m)^{1+1/\varepsilon}\) where \(\varepsilon \in [0, 1]\).\(^{10}\) The version of this model that we use here is similar to the one in Keuschnigg (2005).
With probability $1 - e_{hi}^m$ worker $h$ looses the job in MNE $m$. If so, he gets a job in another MNE at the wage $w_i$ with probability $1 - u_i$, where $u_i \in [0, 1]$ is the unemployment rate. With probability $u_i$ he does not get an alternative job and has income that, without loss of generality, is normalized to zero. The expected income of household $h$ is therefore equal to $e_{hi}^m w_i + (1 - e_{hi}^m)(1 - u_i)w_i$. Maximizing expected income less effort costs yields the optimal effort level $e_{hi}^m = \left[ w_i - (1 - u_i)w_i \right]^{1/\epsilon} =: \tilde{E}(w_i^m, u_i, w_i)$.

The production function of MNE $m$ is now given by $F(k_i^m, x_i^m)$, where $x_i^m := \ell_i^m \tilde{E}(w_i^m, u_i, w_i)$ are effective units of labor. The production function has the same properties with respect to $k_i^m$ and $x_i^m$ as it had in the previous sections with respect to $k_i$ and $\ell_i$. Tax base definitions and the tax rules are also the same as above. Following the basic idea of the efficiency wage approach, the MNE now maximizes after-tax profits not only with respect to capital $k_i^m$, labor input $\ell_i^m$ and profit shifting $s_i^m$, but also with respect to the wage rate $w_i^m$. Under separate accounting, the first-order conditions are $(1 - t_i)(F_k - \rho r) - r(1 - \rho) = 0$, $\tilde{E}F_x - \tilde{w}_i^m = 0$, $\tilde{E}_w^m F_x - 1 = 0$ and $C' = t_a - t_b$, where the tilde over a variable again indicates profit-maximizing values under separate accounting. Analogous conditions hold under formula apportionment, except for additional terms reflecting the derivatives of the apportionment formula $A(\cdot)$.

As all MNEs are identical, the market equilibrium under separate accounting is characterized by $\tilde{\ell}_i = \tilde{\ell}_i$, $\tilde{k}_i = k_i$ and $\tilde{s} = s$. The unemployment rate in country $i$ then equals $u_i = 1 - \int_0^1 \tilde{\ell}_i^m dm = 1 - \tilde{\ell}_i$. Moreover, all MNEs pay the same wage rate in equilibrium, so $\tilde{w}_i^m = w_i =: \tilde{w}_i$ and $\tilde{E}(\cdot) = [\tilde{w}_i(1 - \tilde{\ell}_i)]^{1/\epsilon} =: \tilde{E}(\tilde{w}_i, \tilde{\ell}_i)$. Using these properties in the above first-order conditions of profit maximization immediately yields $\tilde{\ell}_i = 1 - \epsilon =: \hat{\ell}$ and $\tilde{E}(\cdot) = (\epsilon \tilde{w}_i)^{1/\epsilon} =: E(\tilde{w}_i)$. The same results are obtained under formula apportionment, even though the proof is more complex.

Hence, in contrast to the fixed wage approach considered in the previous sections, labor input is now fixed and independent of the tax rates chosen by the countries. However, the wage rate is no longer fixed but influenced by corporate income taxation. An interesting implication of this property is that all results derived in the previous sections qualitatively hold also under the efficiency wage approach. The only difference is that all effects which work through changes in the labor input in the fixed wage approach now work through changes in the wage rates. More specific, analogous to Proposition 1 corporate taxation under separate accounting is characterized by a
positive profit shifting externality and a negative income externality. Under formula apportionment we obtain all externalities as in the fixed wage approach, except for the newly derived unemployment externality. This latter externality is replaced, however, by a so far ignored wage rate externality that has the same properties as listed in Proposition 2. It is maximized under a pure payroll formula and minimized under a pure sales (property formula) if the substitution elasticity is low (high). The sum of the wage rate and income externalities is positive if the production elasticity of labor is high, similar to the insights derived in Proposition 3. Finally, under the Cobb-Douglas specification of the production function the formula externality is minimized for the pure sales formula and the sum of the tax base and formula externalities is positive with all consequences already derived in Propositions 4–7.\footnote{We can prove this last result either by focusing on the pure formulas or by assuming that $\varepsilon$ is not too low. Moreover, we were not able to identify a counterexample.}

We can summarize that turning from the fixed wage approach to the efficiency wage approach changes the channel through which corporate income taxation causes fiscal externalities (wage rates instead of labor input), but the overall efficiency properties of corporate income taxation in the presence of unemployment remain unaffected.\footnote{At the cost of more complexity, we can even generalize the results to the case with risk-averse households, provided relative risk aversion is constant. With non-constant relative risk aversion, fiscal externalities are caused through changes in both labor input and wage rates. However, the analysis then becomes untractable and it might be useful to first investigate the implications of non-constant relative risk aversion in a standard capital tax competition model that does not distinguish between separate accounting and formula apportionment. We leave such an analysis for future research.}

6 Conclusion

This paper analyzed corporate income taxation of MNEs in the presence of labor market imperfections. We used a two country model with multinational activities and involuntary unemployment. While unemployment does not cause a fiscal externality under separate accounting, under formula apportionment we identified an unemployment externality. This externality is positive since an increase in one country’s tax rate improves employment in the other country via a tax base and a formula effect. As intuitively plausible, we showed that the unemployment externality is maximized...
under a pure payroll formula. It is minimized for a pure sales (property) formula if the substitution elasticity is low (high). Moreover, the unemployment externality tends to outweigh other externalities known from previous studies such that tax rates become inefficiently low. The distortions are minimized under a pure sales formula.

Even though we showed that these results are robust with respect to variations in the set of tax instruments and in the modeling approach to unemployment, there are still interesting extensions of our analysis. For example, we followed the previous literature and assumed perfectly identical countries. This is a useful first step in order to understand the basic mechanisms at work. But it might be an interesting task to analyze the implications of country asymmetries for the fiscal externalities arising under separate accounting and formula apportionment. However, before investigating these implications in the presence of unemployment it is important to understand them for perfectly competitive labor markets. To the best of our knowledge, such a study is missing in the literature and, thus, we leave the analysis of corporate income taxation and unemployment in a model with asymmetric countries for future research, too.

Appendix

Derivation of (13a)-(13d). Total differentiating (11a) and (11b) and then applying the symmetry property yields

\[-2\phi A_k (dt_a - dt_b) - (F_k - r\rho) d\tau + (1-t)(F_{kk} d\hat{k}_i + F_{k\ell} d\hat{\ell}_i) = 0, \quad (A1)\]
\[-2\phi A_\ell (dt_a - dt_b) + (1-t)(F_{\ell k} d\hat{k}_i + F_{\ell\ell} d\hat{\ell}_i) = 0. \quad (A2)\]

Inserting \(d\tau = Adt_a + (1-A)dt_b\) and then solving the system of equations with the help of Cramer’s rule yields

\[(1-t)H \cdot d\hat{k}_i = [2\phi A_k (dt_a - dt_b) + (F_k - r\rho) [Adt_a + (1-A)dt_b]] F_{\ell\ell} - 2\phi A_\ell F_{k\ell} (dt_a - dt_b), \quad (A3)\]
\[(1-t)H \cdot d\hat{\ell}_i = -[2\phi A_k (dt_a - dt_b) + (F_k - r\rho) [Adt_a + (1-A)dt_b]] F_{k\ell} + 2\phi A_\ell F_{kk} (dt_a - dt_b), \quad (A4)\]

where \(H := F_{kk} F_{\ell\ell} - F_{k\ell}^2 > 0\). Finally, using (12) we obtain after some rearrangement of terms (13a)-(13d). \(\blacksquare\)
Proof of Proposition 2. We first have to formalize some properties of the production function. By the Euler Theorem, homogeneity of $F$ implies $\mu F = kF_k + \ell F_\ell$ where $\mu$ denotes the degree of homogeneity. Decreasing returns to scale imply $\mu \in [0, 1]$. Moreover, there exists a function $Z(k/\ell)$ with $Z'(k/\ell) > 0$ and $Z''(k/\ell) < 0$ such that

$$F = \ell^\mu Z, \quad F_k = \ell^{\mu-1}Z', \quad F_{kk} = \ell^{\mu-2}Z'', \quad F_{k\ell} = (\mu - 1)\ell^{\mu-2}(Z' - \ell Z'') < 0, \quad (A5)$$

$$F_\ell = \mu \ell^{\mu-1}Z - \ell^{\mu-2}kZ', \quad F_{\ell\ell} = \mu(\mu - 1)\ell^{\mu-2}Z + 2(1 - \mu)\ell^{\mu-3}kZ' + \ell^{\mu-4}k^2Z''. \quad (A6)$$

We can then write

$$kF_{kk} + \ell F_{k\ell} = -(1 - \mu)\ell^{\mu-1}Z' < 0, \quad (A7)$$

$$F_kF_{kk} + F_\ell(kF_{kk} + \ell F_{k\ell}) = -(1 - \mu)\ell^{\mu-2}(Z'^2 - \ell Z'') < 0, \quad (A8)$$

$$F_kF_{k\ell} - F_\ell F_{kk} = -\ell^{\mu-3}[\mu ZZ'' + (1 - \mu)Z'^2] > 0. \quad (A9)$$

The sign of (A9) follows from our assumptions $F_k, F_\ell, F_{k\ell} > 0$ and $F_{kk} < 0$. It implies $\mu ZZ'' + (1 - \mu)Z'^2 < 0$. The technical rate of substitution of $F$ reads

$$\text{TRS} := \frac{dk}{d\ell} \bigg|_{F=F} = -\frac{F_\ell}{F_k} = \frac{k}{\ell} - \frac{\mu}{Z} Z' < 0, \quad (A10)$$

so the substitution elasticity of $F$ can be written as

$$\eta = \frac{d(k/\ell)}{d\text{TRS}} \cdot \frac{k/\ell}{Z'^2} = \frac{\mu}{k \mu ZZ'' + (1 - \mu)Z'^2} \left( \frac{k}{\ell} - \frac{\mu}{Z} Z' \right) > 0. \quad (A11)$$

The sign of (A11) follows from $\mu ZZ'' + (1 - \mu)Z'^2 < 0$ and (A10). From (A11) we obtain

$$\eta - 1 = \frac{\ell}{k \mu ZZ'' + (1 - \mu)Z'^2} \left[ -ZZ' + k \ell \left( Z'^2 - ZZ'' \right) \right]. \quad (A12)$$

With the help of this relation we can write

$$\begin{align*}
(1 - \mu)F_{k\ell} + F_\ell(kF_{kk} + \ell F_{k\ell}) & = (1 - \mu)\ell^{\mu-3}[\mu ZZ'' + (1 - \mu)Z'^2](\eta - 1)/\mu \lesssim 0 \quad \Leftrightarrow \quad \eta \lesssim 1, \quad (A13)
\end{align*}$$

where we used $\mu ZZ'' + (1 - \mu)Z'^2 < 0$ from (A9).

To prove Proposition 2 note that the unemployment externality in (16c) is proportional to the cross-country effect in (13d). Hence, we can focus on the properties of (13d). The impact of $\gamma$, $\sigma$ and $\varphi$ on this cross-country effect can be written as

$$P_\gamma := \frac{\partial}{\partial \gamma} \left( \frac{\partial \ell_j}{\partial t_i} \bigg|_{(\gamma, \sigma, \varphi)} \right) = \frac{\varphi}{2(1-\ell)H} \frac{F_{k\ell}}{k}. \quad (A14)$$
\[ P_\sigma := \frac{\partial}{\partial \sigma} \left( \frac{\partial \hat{l}_j}{\partial t_i} \bigg|_{(\gamma, \sigma, \varphi)} \right) = \frac{\phi}{2(1-t)H} \frac{F_kF_{kl} - F_lF_{kk}}{F}, \quad \text{(A15)} \]

\[ P_\varphi := \frac{\partial}{\partial \varphi} \left( \frac{\partial \hat{l}_j}{\partial t_i} \bigg|_{(\gamma, \sigma, \varphi)} \right) = -\frac{\phi}{2(1-t)H} \frac{F_{kk}F_kF_{kl}}{\ell}. \quad \text{(A16)} \]

Suppose we start at \( \varphi = 1 \), reduce \( \varphi \) and increase \( \gamma \) and/or \( \sigma \). Taking into account \( d\varphi = -d\gamma - d\sigma \) and \( d\gamma, d\sigma \geq 0 \) with strict inequality in at least one of these two relations, the total effect on (13d) is given by

\[ d \left( \frac{\partial \hat{l}_j}{\partial t_i} \bigg|_{(\gamma, \sigma, \varphi)} \right) = (P_\gamma - P_\varphi)d\gamma + (P_\sigma - P_\varphi)d\sigma < 0, \quad \text{(A17)} \]

since (A14)–(A16), (A7), (A8) and \( \mu F = \hat{k}F_k + \hat{\ell}F_{\ell} \) imply

\[ P_\gamma - P_\varphi = \frac{\phi(\hat{k}F_{kk} + \hat{\ell}F_{\ell})}{2k\ell(1-t)H} < 0, \quad \text{(A18)} \]

\[ P_\sigma - P_\varphi = \frac{\phi[(1-\mu)FF_{kk} + F_k(\hat{k}F_{kk} + \hat{\ell}F_{\ell})]}{2\ell(1-t)FH} < 0. \quad \text{(A19)} \]

Hence, deviating from the pure payroll formula reduces the unemployment externality, so \( \text{UE}_{(\gamma, \sigma, \varphi)} \) is maximized at \((\gamma, \sigma, \varphi) = (0, 0, 1)\). This proves part (i) of Proposition 2.

Next suppose we start at \( \sigma = 1 \), reduce \( \sigma \) and increase \( \gamma \) and/or \( \varphi \). This means \( d\sigma = -d\gamma - d\varphi \) and \( d\gamma, d\varphi \geq 0 \). If \( \eta \in [0, 1] \), then the total effect on (13d) equals

\[ d \left( \frac{\partial \hat{l}_j}{\partial t_i} \bigg|_{(\gamma, \sigma, \varphi)} \right) = (P_\sigma - P_\varphi)d\sigma + (P_\varphi - P_\sigma)d\varphi > 0, \quad \text{(A20)} \]

since \( P_\varphi - P_\sigma > 0 \) according to (A19) and since (A13) together with \( \eta \in [0, 1] \) implies

\[ P_\gamma - P_\sigma = \frac{\phi[(1-\mu)FF_{kk} + F_k(\hat{k}F_{kk} + \hat{\ell}F_{\ell})]}{2k(1-\hat{t})FH} > 0. \quad \text{(A21)} \]

Hence, for \( \eta \in [0, 1] \) the unemployment externality is minimized at \((\gamma, \sigma, \varphi) = (0, 1, 0)\). This proves part (ii) of Proposition 2. For \( \eta \in [1, \infty[ \), we start at \( \gamma = 1 \), reduce \( \gamma \) and increase \( \sigma \) and/or \( \varphi \), i.e. \( d\gamma = -d\sigma - d\varphi \) and \( d\sigma, d\varphi \geq 0 \). The total effect on (13d) is

\[ d \left( \frac{\partial \hat{l}_j}{\partial t_i} \bigg|_{(\gamma, \sigma, \varphi)} \right) = (P_\sigma - P_\gamma)d\sigma + (P_\varphi - P_\gamma)d\varphi > 0, \quad \text{(A22)} \]
since \( P_\psi - P_\gamma > 0 \) due to (A18) and \( P_\sigma - P_\gamma > 0 \) due to (A13), (A21) and \( \eta \in [1, \infty[ \). This proves part (iii) of Proposition 2.

It remains to show that the unemployment externality is always positive. For \( \eta \in [0, 1] \), it is minimized at \( \sigma = 1 \). With the help of the definition \( \phi = F - \hat{\ell}F_\ell - \rho r\hat{k} \), the cross-country effect (13d) for \((\gamma, \sigma, \varphi) = (0, 1, 0)\) can be written as

\[
\frac{\partial \hat{\ell}_j}{\partial t_i}\bigg|_{(0, 1, 0)} = \frac{1}{2(1 - \hat{\ell})} H \left\{ -(1 - \mu)F_\ell F_{k\ell} - \frac{F_{k\ell}(\hat{k}F_{kk} + \hat{\ell}F_{k\ell})}{F} \right. \\
+ \left. \rho r(1 - \mu)FF_{k\ell} + F_{\ell}(\hat{k}F_{kk} + \hat{\ell}F_{k\ell}) \right\} > 0. \quad (A23)
\]

The sign of (A23) follows from (A7), (A13) and \( \eta \in [0, 1] \). Due to Proposition 2 (ii), (A23) is the lowest value of the cross-country effect if \( \eta \in [0, 1] \), so the unemployment externality is positive also for all other formulas \((\gamma, \sigma, \varphi) \in S \). The case of \( \eta \in [1, \infty[ \) is analogous. Equation (13d) is then minimized at \((\gamma, \sigma, \varphi) = (1, 0, 0)\). We obtain

\[
\frac{\partial \hat{\ell}_j}{\partial t_i}\bigg|_{(1, 0, 0)} = \frac{(1 - \mu)F_{k\ell}}{2(1 - \hat{\ell})H} > 0. \quad (A24)
\]

This is the minimum of the cross-country effect (13d). The unemployment externality is therefore positive also for all other formulas. \( \square \)

**Proof of Proposition 3.** For \( \eta \in [0, 1] \) Proposition 2 states that the unemployment externality is minimized for the pure sales formula. Hence, in this case \( IE + UE|_{(\gamma, \sigma, \varphi)} > 0 \) for all \((\gamma, \sigma, \varphi) \in S \) if and only if \( IE + UE|_{(0, 1, 0)} > 0 \). Using (11a) (13d), (16a), (16c), \( \phi = F - \hat{\ell}F_\ell - \rho r\hat{k} \), \( \mu F = \hat{k}F_k + \hat{\ell}F_\ell \) and \( \theta = 1/2 \) we obtain

\[
IE + UE|_{(0, 1, 0)} = \frac{\lambda F}{2(1 - \hat{\ell})} \left\{ (1 - \mu)F \left[ \frac{F_{\ell}F_{k\ell} - F_{k\ell}F_{kk}}{F} \right] - (1 - \hat{\ell}) \right. \\
- \left. \frac{(1 - \hat{\ell})(1 - \rho)\hat{k}F_k}{1 - \rho \hat{k}} \left[ 1 + \frac{F_{\ell}}{(1 - \hat{\ell})H} \frac{(1 - \mu)FF_{k\ell} + F_{\ell}(\hat{k}F_{kk} + \hat{\ell}F_{k\ell})}{\hat{k}F} \right] \right\}. \quad (A25)
\]

With the help of (A9), (A13) and \( H = F_{kk}F_{k\ell} - F_{k\ell}^2 = -(1 - \mu)\hat{\ell}^{2\mu-4}[\mu ZZ'' + (1 - \mu)Z'^2], \) equation (A25) can be rewritten as

\[
IE + UE|_{(0, 1, 0)} = \frac{\lambda F}{2(1 - \hat{\ell})} \left\{ (1 - \mu) \left[ \frac{\hat{\ell}F_\ell}{(1 - \mu)F} - (1 - \hat{\ell}) \right] \\
- \frac{(1 - \hat{\ell})(1 - \rho)\hat{k}F_k}{1 - \rho \hat{k}} \left[ 1 + \frac{(1 - \eta)\hat{\ell}F_\ell}{\mu(1 - \hat{\ell})F} \right] \right\}. \quad (A26)
\]
Using \( \beta = \hat{\ell}F_t/F = \mu - \hat{k}F_k/F \), (A26) turns out to be positive if and only if

\[
\frac{\beta^2}{\mu} + \frac{\eta(2 - t - \rho) + (1 - \eta)(1 - \tilde{\ell})}{(1 - \eta)(1 - \rho)} \beta - \frac{(1 - \hat{\ell})(1 - \rho)(1 - \rho \hat{\ell}) + \mu(1 - \rho)}{(1 - \eta)(1 - \rho)} > 0. \tag{A27}
\]

The solution to this quadratic inequality is \( \beta > \bar{\beta} \) and \( \beta < \underline{\beta} \), where \( \bar{\beta} \) is the RHS of (17) and \( \underline{\beta} \) is the same expression except for replacing the minus sign in front of the square root by a plus sign. It is straightforward to show that \( \underline{\beta} < 0 \), so only the condition \( \beta > \bar{\beta} \) is relevant. This proves part (i) of Proposition 3.

To prove part (ii) note that for \( \eta \in [1, \infty[ \) the unemployment externality is minimized for a pure property formula according to Proposition 2. In this case we obtain

\[
IE + UE|_{(1,0,0)} = \frac{\lambda}{2(1 - \hat{\ell})} \left\{ (1 - \mu)F \left[ \frac{F_k k_{k\ell}}{H} - (1 - \hat{\ell}) \right] - \frac{(1 - \hat{\ell})(1 - \rho)}{1 - \rho \hat{\ell}} kF_k \right\}. \tag{A28}
\]

Using (A11), \( \beta = \hat{\ell}F_t/F = \mu - \hat{k}F_k/F \) and \( (1 - \mu)F F_{k\ell}/(\hat{k}H) = [1 - \eta(1 - \mu)]/\mu \), it turns out that (A28) is positive if and only if (18) is satisfied. \[ \blacksquare \]

**Proof of Proposition 4.** Inserting (13a)–(13d) and \( \theta = 1/2 \) into (16b) yields

\[
\text{FE}|_{(\gamma, \sigma, \varphi)} = \frac{i \phi^2 V'}{2(1 - \hat{\ell})H} \left\{ - \frac{\gamma^2 F_{k\ell}}{k^2} - \frac{\sigma^2 (F_k^2 F_{k\ell} - 2 F_k F_{k\ell} F_{k\ell} + F_{k\ell}^2 F_{kk})}{F^2} - \frac{\varphi^2 F_{kk}}{k^2} \right. \\
+ \frac{2 \gamma \varphi F_{k\ell}}{k F} + \frac{2 \gamma \sigma (F_k F_{k\ell} - F_{k\ell} F_{k\ell})}{k F} - \frac{2 \sigma \varphi (F_{k\ell} F_{k\ell} - F_{k\ell} F_{k\ell})}{F^2} \right\} > 0. \tag{A29}
\]

Differentiating (A29) with respect to the formula weights, we obtain

\[
R_\gamma := \frac{\partial \text{FE}|_{(\gamma, \sigma, \varphi)}}{\partial \gamma} = \frac{i \phi^2 V'}{2(1 - \hat{\ell})H} \left[ - \frac{\gamma F_{k\ell}}{k^2} + \frac{\varphi F_{k\ell}}{kF} - \frac{\sigma (F_k F_{k\ell} - F_{k\ell} F_{k\ell})}{k F} \right], \quad \tag{A30}
\]

\[
R_\sigma := \frac{\partial \text{FE}|_{(\gamma, \sigma, \varphi)}}{\partial \sigma} = \frac{i \phi^2 V'}{2(1 - \hat{\ell})H} \left[ - \frac{\sigma (F_k^2 F_{k\ell} - 2 F_k F_{k\ell} F_{k\ell} + F_{k\ell}^2 F_{kk})}{F^2} \right. \\
- \frac{\gamma (F_k F_{k\ell} - F_{k\ell} F_{k\ell})}{kF} - \frac{\varphi (F_{k\ell} F_{k\ell} - F_{k\ell} F_{k\ell})}{F^2} \left. \right] \tag{A31}
\]

\[
R_\varphi := \frac{\partial \text{FE}|_{(\gamma, \sigma, \varphi)}}{\partial \varphi} = \frac{i \phi^2 V'}{2(1 - \hat{\ell})H} \left[ - \frac{\varphi F_{k\ell}}{k^2} + \frac{\gamma F_{k\ell}}{kF} - \frac{\sigma (F_{k\ell} F_{k\ell} - F_{k\ell} F_{k\ell})}{k F} \right]. \tag{A32}
\]

Now start at \( \sigma = 1 \), reduce \( \sigma \) and increase \( \gamma \) and/or \( \varphi \). Using \( \text{d} \sigma = - \text{d} \gamma - \text{d} \varphi \), the total effect on the formula externality can be written as

\[
\text{d}\text{FE}|_{(\gamma, \sigma, \varphi)} = (R_\gamma - R_\sigma) \text{d} \gamma + (R_\varphi - R_\sigma) \text{d} \varphi. \tag{A33}
\]
The Cobb-Douglas specification implies \( F_k = \alpha \hat{k}^{\alpha-1} \hat{\ell}^\beta \), \( F_\ell = \beta \hat{k}^\alpha \hat{\ell}^{\beta-1} \), \( F_{kk} = \alpha (\alpha - 1) \hat{k}^{\alpha-2} \hat{\ell}^\beta \) and \( F_{\ell\ell} = \beta (\beta - 1) \hat{k}^\alpha \hat{\ell}^{\beta-2} \). Using (A30)–(A32) then yields

\[
R_\gamma - R_\sigma = \beta (1 - \alpha - \beta) \left( \gamma + \alpha \sigma \right) \frac{\hat{t} \phi^2 \hat{k}^{\alpha-2} \hat{\ell}^{\beta-2} V'}{(1 - \hat{t}) H} > 0,
\]

\[
R_\phi - R_\sigma = \alpha (1 - \alpha - \beta) \left( \phi + \beta \sigma \right) \frac{\hat{t} \phi^2 \hat{k}^{\alpha-2} \hat{\ell}^{\beta-2} V'}{(1 - \hat{t}) H} > 0.
\]

Using this and \( d\gamma, d\phi \geq 0 \) in (A33) implies \( d\text{FE} > 0 \). Hence, the formula externality is minimized for the pure sales formula, as stated Proposition 4 (i).

The proof of part (ii) is trivial for \( \rho = 1 \), since then \( \text{TE} = 0 \). For \( \rho = 0 \) we obtain \( \phi = F - \ell F_\ell = (1 - \beta) \hat{k}^\alpha \hat{\ell}^\beta \) and

\[
\text{TE} + \text{FE}\big|_{(0,1,0)} = \alpha \beta^2 (1 - \beta) (1 - \alpha - \beta) \frac{\hat{t} \phi^2 \hat{k}^{\alpha-2} \hat{\ell}^{\beta-2} V'}{2(1 - \hat{t}) H} > 0. \tag{A34}
\]

Since for all other formulas the formula externality is higher than for the pure sales formula, the sign of (A34) is true for all convex combinations of \( \gamma, \sigma \) and \( \phi \). ■

References


